In previous courses and in Chapters 1–6, you learned the following skills, which you’ll use in Chapter 7: classifying triangles, simplifying radicals, and solving proportions.

**Prerequisite Skills**

**VOCABULARY CHECK**
Name the triangle shown.

1. 

2. 

3. 

4. 

**SKILLS AND ALGEBRA CHECK**
Simplify the radical. *(Review p. 874 for 7.1, 7.2, 7.4.)* 

5. \( \sqrt{45} \)

6. \( (3\sqrt{7})^2 \)

7. \( \sqrt{3} \cdot \sqrt{5} \)

8. \( \frac{7}{\sqrt{2}} \)

Solve the proportion. *(Review p. 356 for 7.3, 7.5–7.7.)* 

9. \( \frac{3}{x} = \frac{12}{16} \)

10. \( \frac{2}{3} = \frac{x}{18} \)

11. \( \frac{x + 5}{4} = \frac{1}{2} \)

12. \( \frac{x + 4}{x - 4} = \frac{6}{5} \)
In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 493. You will also use the key vocabulary listed below.

**Big Ideas**

1. Using the Pythagorean Theorem and its converse
2. Using special relationships in right triangles
3. Using trigonometric ratios to solve right triangles

**KEY VOCABULARY**

- Pythagorean triple, p. 435
- trigonometric ratio, p. 466
- tangent, p. 466
- sine, p. 473
- cosine, p. 473
- angle of elevation, p. 475
- angle of depression, p. 475
- solve a right triangle, p. 483
- inverse tangent, p. 483
- inverse sine, p. 483
- inverse cosine, p. 483

You can use trigonometric ratios to find unknown side lengths and angle measures in right triangles. For example, you can find the length of a ski slope.

**Animated Geometry**

The animation illustrated below for Example 4 on page 475 helps you answer this question: How far will you ski down the mountain?

You can use right triangles to find the distance you ski down a mountain.

Click on the “Spin” button to generate values for y and z. Find the value of x.

**Why?**

Other animations for Chapter 7: pages 434, 442, 450, 460, and 462
Chapter 7  Right Triangles and Trigonometry

**QUESTION**

What relationship exists among the sides of a right triangle?

Recall that a square is a four sided figure with four right angles and four congruent sides.

**EXPLORE**  Make and use a tangram set

**STEP 1** Make a tangram set  On your graph paper, copy the tangram set as shown. Label each piece with the given letters. Cut along the solid black lines to make seven pieces.

**STEP 2** Trace a triangle  On another piece of paper, trace one of the large triangles $P$ of the tangram set.

**STEP 3** Assemble pieces along the legs  Use all of the tangram pieces to form two squares along the legs of your triangle so that the length of each leg is equal to the side length of the square. Trace all of the pieces.

**STEP 4** Assemble pieces along the hypotenuse  Use all of the tangram pieces to form a square along the hypotenuse so that the side length of the square is equal to the length of the hypotenuse. Trace all of the pieces.

**DRAW CONCLUSIONS**  Use your observations to complete these exercises

1. Find the sum of the areas of the two squares formed in Step 3. Let the letters labeling the figures represent the area of the figure. How are the side lengths of the squares related to Triangle $P$?

2. Find the area of the square formed in Step 4. How is the side length of the square related to Triangle $P$?

3. Compare your answers from Exercises 1 and 2. Make a conjecture about the relationship between the legs and hypotenuse of a right triangle.

4. The triangle you traced in Step 2 is an isosceles right triangle. Why? Do you think that your conjecture is true for all isosceles triangles? Do you think that your conjecture is true for all right triangles? Justify your answers.
7.1 Apply the Pythagorean Theorem

**Before**
You learned about the relationships within triangles.

**Now**
You will find side lengths in right triangles.

**Why?**
So you can find the shortest distance to a campfire, as in Ex. 35.

Key Vocabulary
- Pythagorean triple
- right triangle, p. 217
- leg of a right triangle, p. 241
- hypotenuse, p. 241

One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras (around 500 B.C.). This theorem can be used to find information about the lengths of the sides of a right triangle.

**THEOREM**

**For Your Notebook**

**THEOREM 7.1 Pythagorean Theorem**
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

*Proof:* p. 434; Ex. 32, p. 455

\[ c^2 = a^2 + b^2 \]

---

**Example 1** Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.

**Solution**

\[(hypotenuse)^2 = (leg)^2 + (leg)^2\]

\[ x^2 = 6^2 + 8^2 \]

\[ x^2 = 36 + 64 \]

\[ x^2 = 100 \]

\[ x = 10 \]

Find the positive square root.

---

**Guided Practice** for Example 1

Identify the unknown side as a leg or hypotenuse. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

1. \[ \frac{x}{3} \]

2. \[ \frac{x}{4} \]
**Example 2** Standardized Test Practice

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?

- A 240 feet
- B 20 feet
- C 16.5 feet
- D 15.5 feet

**Solution**

\[
\text{Length of ladder}^2 = \text{Distance from house}^2 + \text{Height of ladder}^2
\]

\[16^2 = 4^2 + x^2 \quad \text{Substitute.}
\]

\[256 = 16 + x^2 \quad \text{Multiply.}
\]

\[240 = x^2 \quad \text{Subtract 16 from each side.}
\]

\[\sqrt{240} = x \quad \text{Find positive square root.}
\]

\[15.491 \approx x \quad \text{Approximate with a calculator.}
\]

The ladder is resting against the house at about 15.5 feet above the ground.

The correct answer is D.  (A) (B) (C) (D)

---

**Guided Practice** for Example 2

3. The top of a ladder rests against a wall, 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is the length of the ladder?

4. The Pythagorean Theorem is only true for what type of triangle?

---

**Proving the Pythagorean Theorem** There are many proofs of the Pythagorean Theorem. An informal proof is shown below. You will write another proof in Exercise 32 on page 455.

In the figure at the right, the four right triangles are congruent, and they form a small square in the middle. The area of the large square is equal to the area of the four triangles plus the area of the smaller square.

\[
(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2
\]

Use area formulas.

\[a^2 + 2ab + b^2 = 2ab + c^2 \quad \text{Multiply.}
\]

\[a^2 + b^2 = c^2 \quad \text{Subtract 2ab from each side.}
\]
**Example 3** Find the area of an isosceles triangle

Find the area of the isosceles triangle with side lengths 10 meters, 13 meters, and 13 meters.

**Solution**

**STEP 1** Draw a sketch. By definition, the length of an altitude is the height of a triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.

**STEP 2** Use the Pythagorean Theorem to find the height of the triangle.

\[ c^2 = a^2 + b^2 \]  
Pythagorean Theorem

\[ 13^2 = 5^2 + h^2 \]  
Substitute.

\[ 169 = 25 + h^2 \]  
Multiply.

\[ 144 = h^2 \]  
Subtract 25 from each side.

\[ 12 = h \]  
Find the positive square root.

**STEP 3** Find the area.

\[ \text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(12) = 60 \text{ m}^2 \]

The area of the triangle is 60 square meters.

**Guided Practice** for Example 3

Find the area of the triangle.

5. \[ \begin{array}{c}
18 \text{ ft} \\
30 \text{ ft}
\end{array} \]

6. \[ \begin{array}{c}
20 \text{ m} \\
26 \text{ m}
\end{array} \]

**Pythagorean Triples** A **Pythagorean triple** is a set of three positive integers \(a, b,\) and \(c\) that satisfy the equation \(c^2 = a^2 + b^2.\)

**Key Concept**

<table>
<thead>
<tr>
<th>Common Pythagorean Triples and Some of Their Multiples</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 4, 5</td>
</tr>
<tr>
<td>6, 8, 10</td>
</tr>
<tr>
<td>9, 12, 15</td>
</tr>
<tr>
<td>30, 40, 50</td>
</tr>
<tr>
<td>3x, 4x, 5x</td>
</tr>
</tbody>
</table>

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.
EXAMPLE 4 Find the length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.

Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 5, 12, 13. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: \( 5 \cdot 2 = 10 \) and \( 12 \cdot 2 = 24 \). So, the length of the hypotenuse is \( 13 \cdot 2 = 26 \).

Method 2: Use the Pythagorean Theorem.

\[
x^2 = 10^2 + 24^2 \quad \text{Pythagorean Theorem}
\]
\[
x^2 = 100 + 576
\]
\[
x^2 = 676
\]
\[
x = 26 \quad \text{Find the positive square root.}
\]

GUIDED PRACTICE for Example 4

Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.

7. \( x \) in.
8. \( x \) cm

7.1 EXERCISES

1. VOCABULARY Copy and complete: A set of three positive integers \( a, b, \) and \( c \) that satisfy the equation \( c^2 = a^2 + b^2 \) is called a ___.

2. ★ WRITING Describe the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle.

3. ALGEBRA Find the length of the hypotenuse of the right triangle.

3. \( x \) with sides 50 and 120.
4. \( x \) with sides 56 and 33.
5. \( x \) with sides 42 and 40.
ERROR ANALYSIS  Describe and correct the error in using the Pythagorean Theorem.

6. \[ \begin{align*}
26^2 + 10^2 &= 24^2 \\
676 + 100 &= 576 \quad \text{X}
\end{align*} \]

7. \[ \begin{align*}
x^2 &= 7^2 + 24^2 \\
x^2 &= 49 + 576 \\
x^2 &= 625 \\
x &= 25 \quad \text{X}
\end{align*} \]

FINDING A LENGTH  Find the unknown leg length \(x\).

8. \[ \begin{align*}
x &= \sqrt{16.7^2 - 8.9^2} \\
x &= \sqrt{289 - 79.21} \\
x &= \sqrt{209.79} \\
x &\approx 14.47 \text{ ft}
\end{align*} \]

9. \[ \begin{align*}
x &= \sqrt{9.8^2 - 4.9^2} \\
x &= \sqrt{96.04 - 24.01} \\
x &= \sqrt{72.03} \\
x &\approx 8.48 \text{ in.}
\end{align*} \]

10. \[ \begin{align*}
x &= \sqrt{6.7^2 - 4.9^2} \\
x &= \sqrt{44.89 - 24.01} \\
x &= \sqrt{20.88} \\
x &\approx 4.54 \text{ ft}
\end{align*} \]

FINDING THE AREA  Find the area of the isosceles triangle.

11. \[ \begin{align*}
\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\
\text{Area} &= \frac{1}{2} \times 17 \times 16 \\
\text{Area} &= 136 \text{ m}^2
\end{align*} \]

12. \[ \begin{align*}
\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\
\text{Area} &= \frac{1}{2} \times 20 \times 12 \\
\text{Area} &= 120 \text{ ft}^2
\end{align*} \]

13. \[ \begin{align*}
\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\
\text{Area} &= \frac{1}{2} \times 10 \times 10 \\
\text{Area} &= 50 \text{ cm}^2
\end{align*} \]

FINDING SIDE LENGTHS  Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.

14. \[ \begin{align*}
x &= \sqrt{72^2 - 21^2} \\
x &= \sqrt{5184 - 441} \\
x &= \sqrt{4743} \\
x &\approx 69 \text{ ft}
\end{align*} \]

15. \[ \begin{align*}
x &= \sqrt{50^2 - 30^2} \\
x &= \sqrt{2500 - 900} \\
x &= \sqrt{1600} \\
x &= 40 \text{ ft}
\end{align*} \]

16. \[ \begin{align*}
x &= \sqrt{60^2 - 39^2} \\
x &= \sqrt{3600 - 1521} \\
x &= \sqrt{2079} \\
x &\approx 45 \text{ ft}
\end{align*} \]

17. ★ MULTIPLE CHOICE  What is the length of the hypotenuse of a right triangle with leg lengths of 8 inches and 15 inches?

A  13 inches  B  17 inches  C  21 inches  D  25 inches

PYTHAGOREAN TRIPLES  The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.

18. 24 and 51  19. 20 and 25  20. 28 and 96

21. 20 and 48  22. 75 and 85  23. 72 and 75
FINDING SIDE LENGTHS Find the unknown side length \( x \). Write your answer in simplest radical form.

24. 

25. 

26. 

27. ★ MULTIPLE CHOICE What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet?

- A \( 270 \text{ ft}^2 \)
- B \( 292.5 \text{ ft}^2 \)
- C \( 540 \text{ ft}^2 \)
- D \( 585 \text{ ft}^2 \)

28. ★ ALGEBRA Solve for \( x \) if the lengths of the two legs of a right triangle are \( 2x \) and \( 2x + 4 \), and the length of the hypotenuse is \( 4x - 4 \).

CHALLENGE In Exercises 29 and 30, solve for \( x \).

29. 

30. 

31. BASEBALL DIAMOND In baseball, the distance of the paths between each pair of consecutive bases is 90 feet and the paths form right angles. How far does the ball need to travel if it is thrown from home plate directly to second base?

32. APPLE BALLOON You tie an apple balloon to a stake in the ground. The rope is 10 feet long. As the wind picks up, you observe that the balloon is now 6 feet away from the stake. How far above the ground is the balloon now?

33. ★ SHORT RESPONSE Three side lengths of a right triangle are 25, 65, and 60. Explain how you know which side is the hypotenuse.

34. MULTI-STEP PROBLEM In your town, there is a field that is in the shape of a right triangle with the dimensions shown.
   a. Find the perimeter of the field.
   b. You are going to plant dogwood seedlings about every ten feet around the field's edge. How many trees do you need?
   c. If each dogwood seedling sells for $12, how much will the trees cost?
35. **MULTIPLE REPRESENTATIONS** As you are gathering leaves for a science project, you look back at your campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.

![Diagram](image)

a. **Making a Table** Make a table with columns labeled \(BC\), \(AC\), \(CE\), and \(AC + CE\). Enter values of \(BC\) from 10 to 120 in increments of 10.

b. **Calculating Values** Calculate \(AC\), \(CE\), and \(AC + CE\) for each value of \(BC\), and record the results in the table. Then, use your table of values to determine the shortest distance you must travel.

c. **Drawing a Picture** Draw an accurate picture to scale of the shortest distance.

36. ★ **SHORT RESPONSE** Justify the Distance Formula using the Pythagorean Theorem.

37. **PROVING THEOREM 4.5** Find the Hypotenuse-Leg (HL) Congruence Theorem on page 241. Assign variables for the side lengths in the diagram. Use your variables to write GIVEN and PROVE statements. Use the Pythagorean Theorem and congruent triangles to prove Theorem 4.5.

38. **CHALLENGE** Trees grown for sale at nurseries should stand at least five feet from one another while growing. If the trees are grown in parallel rows, what is the smallest allowable distance between rows?
7.2 Converse of the Pythagorean Theorem

**MATERIALS** - graphing calculator or computer

**QUESTION** How can you use the side lengths in a triangle to classify the triangle by its angle measures?

You can use geometry drawing software to construct and measure triangles.

**EXPLORE Construct a triangle**

**STEP 1 Draw a triangle** Draw any \( \triangle ABC \) with the largest angle at \( C \). Measure \( \angle C, AB, AC, \) and \( CB \).

**STEP 2 Calculate** Use your measurements to calculate \( AB^2, AC^2, CB^2, \) and \( (AC^2 + CB^2) \).

**STEP 3 Complete a table** Copy the table below and record your results in the first row. Then move point \( A \) to different locations and record the values for each triangle in your table. Make sure \( AB \) is always the longest side of the triangle. Include triangles that are acute, right, and obtuse.

<table>
<thead>
<tr>
<th>( m\angle C )</th>
<th>( AB )</th>
<th>( AB^2 )</th>
<th>( AC )</th>
<th>( CB )</th>
<th>( AC^2 + CB^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>76°</td>
<td>5.2</td>
<td>27.04</td>
<td>4.5</td>
<td>3.8</td>
<td>34.69</td>
</tr>
</tbody>
</table>

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. The Pythagorean Theorem states that “In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.” Write the Pythagorean Theorem in if-then form. Then write its converse.

2. Is the converse of the Pythagorean Theorem true? Explain.

3. Make a conjecture about the relationship between the measure of the largest angle in a triangle and the squares of the side lengths.

**Copy and complete the statement.**

4. If \( AB^2 > AC^2 + CB^2 \), then the triangle is a(n) __?__ triangle.

5. If \( AB^2 < AC^2 + CB^2 \), then the triangle is a(n) __?__ triangle.

6. If \( AB^2 = AC^2 + CB^2 \), then the triangle is a(n) __?__ triangle.
7.2 Use the Converse of the Pythagorean Theorem

Before
You used the Pythagorean Theorem to find missing side lengths.

Now
You will use its converse to determine if a triangle is a right triangle.

Why?
So you can determine if a volleyball net is set up correctly, as in Ex. 38.

Key Vocabulary
- acute triangle, p. 217
- obtuse triangle, p. 217

The converse of the Pythagorean Theorem is also true. You can use it to verify that a triangle with given side lengths is a right triangle.

THEOREM
THEOREM 7.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

If \( c^2 = a^2 + b^2 \), then \( \triangle ABC \) is a right triangle.

Proof: Ex. 42, p. 446

EXAMPLE 1 Verify right triangles

Tell whether the given triangle is a right triangle.

a. \( \sqrt{34} \)

b. \( \sqrt{14} \)

Let \( c \) represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation \( c^2 = a^2 + b^2 \).

a. \( (3\sqrt{34})^2 = 9^2 + 15^2 \)

\( 9 \cdot 34 = 81 + 225 \)

\( 306 \neq 306 \)

The triangle is not a right triangle.

b. \( 26^2 = 22^2 + 14^2 \)

\( 676 \neq 484 + 196 \)

\( 676 \neq 680 \)

The triangle is a right triangle.

REVIEW ALGEBRA
Use a square root table or a calculator to find the decimal representation. So, \( 3\sqrt{34} \approx 17.493 \) is the length of the longest side in part (a).

GUIDED PRACTICE for Example 1

Tell whether a triangle with the given side lengths is a right triangle.

1. 4, 4\sqrt{3}, 8
2. 10, 11, and 14
3. 5, 6, and \( \sqrt{61} \)
**Example 2** Classify triangles

Can segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle? If so, would the triangle be **acute, right, or obtuse**?

**Solution**

**Step 1** Use the Triangle Inequality Theorem to check that the segments can make a triangle.

4.3 + 5.2 = 9.5
4.3 + 6.1 = 10.4
5.2 + 6.1 = 11.3

9.5 > 6.1 10.4 > 5.2 11.3 > 4.3

> The side lengths 4.3 feet, 5.2 feet, and 6.1 feet can form a triangle.

**Step 2** Classify the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

\[ c^2 \quad ? \quad a^2 + b^2 \]

\[ 6.1^2 \quad ? \quad 4.3^2 + 5.2^2 \]

\[ 37.21 \quad ? \quad 18.49 + 27.04 \]

\[ 37.21 < 45.53 \]

> The side lengths 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

---

**Theorems for Your Notebook**

**Theorem 7.3**

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle \(ABC\) is an acute triangle.

If \(c^2 < a^2 + b^2\), then the triangle \(ABC\) is acute.

*Proof:* Ex. 40, p. 446

**Theorem 7.4**

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle \(ABC\) is an obtuse triangle.

If \(c^2 > a^2 + b^2\), then triangle \(ABC\) is obtuse.

*Proof:* Ex. 41, p. 446

---

**Classifying Triangles**

The Converse of the Pythagorean Theorem is used to verify that a given triangle is a right triangle. The theorems below are used to verify that a given triangle is acute or obtuse.
EXAMPLE 3 Use the Converse of the Pythagorean Theorem

CATAMARAN You are part of a crew that is installing the mast on a catamaran. When the mast is fastened properly, it is perpendicular to the trampoline deck. How can you check that the mast is perpendicular using a tape measure?

Solution

To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

Think of the mast as a line and the deck as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the mast is perpendicular to different lines on the deck.

First place a mark 3 feet up the mast and a mark on the deck 4 feet from the mast. Use the tape measure to check that the distance between the two marks is 5 feet. The mast makes a right angle with the line on the deck.

Finally, repeat the procedure to show that the mast is perpendicular to another line on the deck.

GUIDED PRACTICE for Example 2 and 3

4. Show that segments with lengths 3, 4, and 6 can form a triangle and classify the triangle as acute, right, or obtuse.

5. WHAT IF? In Example 3, could you use triangles with side lengths 2, 3, and 4 to verify that you have perpendicular lines? Explain.

CLASSIFYING TRIANGLEs You can use the theorems from this lesson to classify a triangle as acute, right, or obtuse based on its side lengths.

CONCEPT SUMMARY For Your Notebook

Methods for Classifying a Triangle by Angles Using its Side Lengths

<table>
<thead>
<tr>
<th>Theorem 7.2</th>
<th>Theorem 7.3</th>
<th>Theorem 7.4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /> If $c^2 = a^2 + b^2$, then $m \angle C = 90^\circ$ and $\triangle ABC$ is a right triangle.</td>
<td><img src="image2" alt="Diagram" /> If $c^2 &lt; a^2 + b^2$, then $m \angle C &lt; 90^\circ$ and $\triangle ABC$ is an acute triangle.</td>
<td><img src="image3" alt="Diagram" /> If $c^2 &gt; a^2 + b^2$, then $m \angle C &gt; 90^\circ$ and $\triangle ABC$ is an obtuse triangle.</td>
</tr>
</tbody>
</table>
1. **VOCABULARY** What is the longest side of a right triangle called?

2. **WRITING** Explain how the side lengths of a triangle can be used to classify it as acute, right, or obtuse.

### VERIFYING RIGHT TRIANGLES

Tell whether the triangle is a right triangle.

3. \(\sqrt{72}, 97, 65\)  
4. \(\sqrt{23}, 21.2, 11.4\)  
5. \(2, 6, 5\)  
6. \(14, 10, 4, 19\)  
7. \(5, 26, \sqrt{26}\)  
8. \(89, 80, 39, 23\)

### VERIFYING RIGHT TRIANGLES

Tell whether the given side lengths of a triangle can represent a right triangle.

9. 9, 12, and 15  
10. 9, 10, and 15  
11. 36, 48, and 60  
12. 6, 10, and 2\(\sqrt{34}\)  
13. 7, 14, and 7\(\sqrt{5}\)  
14. 10, 12, and 20

### CLASSIFYING TRIANGLES

In Exercises 15–23, decide if the segment lengths form a triangle. If so, would the triangle be acute, right, or obtuse?

15. 10, 11, and 14  
16. 10, 15, and 5\(\sqrt{13}\)  
17. 24, 30, and 6\(\sqrt{43}\)  
18. 5, 6, and 7  
19. 12, 16, and 20  
20. 8, 10, and 12  
21. 15, 20, and 36  
22. 6, 8, and 10  
23. 8.2, 4.1, and 12.2

24. **MULTIPLE CHOICE** Which side lengths do not form a right triangle?
   - A) 5, 12, 13  
   - B) 10, 24, 28  
   - C) 15, 36, 39  
   - D) 50, 120, 130

25. **MULTIPLE CHOICE** What type of triangle has side lengths of 4, 7, and 9?
   - A) Acute scalene  
   - B) Right scalene  
   - C) Obtuse scalene  
   - D) None of the above

26. **ERROR ANALYSIS** A student tells you that if you double all the sides of a right triangle, the new triangle is obtuse. Explain why this statement is incorrect.

### GRAPHING TRIANGLES

Graph points \(A, B, \) and \(C\). Connect the points to form \(\triangle ABC\). Decide whether \(\triangle ABC\) is acute, right, or obtuse.

27. \(A(-2, 4), B(6, 0), C(-5, -2)\)  
28. \(A(0, 2), B(5, 1), C(1, -1)\)
29. **ALGEBRA** Tell whether a triangle with side lengths $5x$, $12x$, and $13x$ (where $x > 0$) is **acute**, **right**, or **obtuse**.

**USING DIAGRAMS** In Exercises 30 and 31, copy and complete the statement with $<$, $>$, or $\geq$, if possible. If it is not possible, explain why.

30. $m\angle A \ ? \ m\angle D$

31. $m\angle B + m\angle C \ ? \ m\angle E + m\angle F$

32. **OPEN-ENDED MATH** The side lengths of a triangle are 6, 8, and $x$ (where $x > 0$). What are the values of $x$ that make the triangle a right triangle? an acute triangle? an obtuse triangle?

33. **ALGEBRA** The sides of a triangle have lengths $x$, $x + 4$, and 20. If the length of the longest side is 20, what values of $x$ make the triangle acute?

34. **CHALLENGE** The sides of a triangle have lengths $4x + 6$, $2x + 1$, and $6x - 1$. If the length of the longest side is $6x - 1$, what values of $x$ make the triangle obtuse?

**EXAMPLE 3**

**35. PAINTING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are $90^\circ$?

**36. WALKING** You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? **Explain**.

**37. MULTI-STEP PROBLEM** Use the diagram shown.

a. Find $BC$.

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle.

c. Draw and label a similar diagram where $\triangle DBC$ remains a right triangle, but $\triangle ABC$ is not.
38. ★ SHORT RESPONSE You are setting up a volleyball net. To stabilize the pole, you tie one end of a rope to the pole 7 feet from the ground. You tie the other end of the rope to a stake that is 4 feet from the pole. The rope between the pole and stake is about 8 feet 4 inches long. Is the pole perpendicular to the ground? Explain. If it is not, how can you fix it?

39. ★ EXTENDED RESPONSE You are considering buying a used car. You would like to know whether the frame is sound. A sound frame of the car should be rectangular, so it has four right angles. You plan to measure the shadow of the car on the ground as the sun shines directly on the car.

a. You make a triangle with three tape measures on one corner. It has side lengths 12 inches, 16 inches, and 20 inches. Is this a right triangle? Explain.

b. You make a triangle on a second corner with side lengths 9 inches, 12 inches, and 18 inches. Is this a right triangle? Explain.

c. The car owner says the car was never in an accident. Do you believe this claim? Explain.

40. PROVING THEOREM 7.3 Copy and complete the proof of Theorem 7.3.

**GIVEN** In \( \triangle ABC \), \( c^2 < a^2 + b^2 \) where \( c \) is the length of the longest side.

**PROVE** \( \triangle ABC \) is an acute triangle.

**Plan for Proof** Draw right \( \triangle PQR \) with side lengths \( a \), \( b \), and \( x \), where \( \angle R \) is a right angle and \( x \) is the length of the longest side. Compare lengths \( c \) and \( x \).

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
</table>
| 1. In \( \triangle ABC \), \( c^2 < a^2 + b^2 \) where \( c \) is the length of the longest side. In \( \triangle PQR \), \( \angle R \) is a right angle. | 1. \( c^2 < a^2 + b^2 \) \( \Rightarrow \) \( c \) is the longest side. \( \Rightarrow \) \( \angle R \) is a right angle.
| 2. \( a^2 + b^2 = x^2 \) | 2. \( \triangle PQR \) is a right triangle.
| 3. \( c^2 < x^2 \) | 3. \( c \) is the longest side.
| 4. \( c < x \) | 4. A property of square roots
| 5. \( m\angle R = 90^\circ \) | 5. \( m\angle R = 90^\circ \)
| 6. \( m\angle C < m\angle ? \) | 6. Converse of the Hinge Theorem
| 7. \( m\angle C < 90^\circ \) | 7. \( m\angle C < 90^\circ \)
| 8. \( \angle C \) is an acute angle. | 8. \( \angle C \) is an acute angle.
| 9. \( \triangle ABC \) is an acute triangle. | 9. \( \triangle ABC \) is an acute triangle.

41. PROVING THEOREM 7.4 Prove Theorem 7.4. Include a diagram and GIVEN and PROVE statements. (Hint: Look back at Exercise 40.)

42. PROVING THEOREM 7.2 Prove the Converse of the Pythagorean Theorem.

**GIVEN** In \( \triangle LMN \), \( \overline{LM} \) is the longest side, and \( c^2 = a^2 + b^2 \).

**PROVE** \( \triangle LMN \) is a right triangle.

**Plan for Proof** Draw right \( \triangle PQR \) with side lengths \( a \), \( b \), and \( x \). Compare lengths \( c \) and \( x \).
43. ★ SHORT RESPONSE Explain why \( \angle D \) must be a right angle.

44. COORDINATE PLANE Use graph paper.
   a. Graph \( \triangle ABC \) with \( A(-7, 2) \), \( B(0, 1) \) and \( C(-4, 4) \).
   b. Use the slopes of the sides of \( \triangle ABC \) to determine whether it is a right triangle. Explain.
   c. Use the lengths of the sides of \( \triangle ABC \) to determine whether it is a right triangle. Explain.
   d. Did you get the same answer in parts (b) and (c)? If not, explain why.

45. CHALLENGE Find the values of \( x \) and \( y \).

---

MIXED REVIEW

PREVIEW Prepare for Lesson 7.3 in Exs. 46–48.

In Exercises 46–48, copy the triangle and draw one of its altitudes. (p. 319)

46. 

47. 

48. 

Copy and complete the statement. (p. 364)

49. If \( \frac{x}{15} = \frac{3}{7} \) then \( \frac{x}{y} = ? \).  
50. If \( \frac{x}{15} = \frac{3}{2} \) then \( \frac{x}{y} = ? \).  
51. If \( \frac{x}{8} = \frac{2}{9} \) then \( \frac{x+8}{8} = ? \).

52. The perimeter of a rectangle is 135 feet. The ratio of the length to the width is 8 : 1. Find the length and the width. (p. 372)

---

QUIZ for Lessons 7.1–7.2

Find the unknown side length. Write your answer in simplest radical form. (p. 433)

1. 

2. 

3. 

Classify the triangle formed by the side lengths as acute, right, or obtuse. (p. 441)

4. 6, 7, and 9  
5. 10, 12, and 16  
6. 8, 16, and 8\( \sqrt{6} \)  
7. 20, 21, and 29  
8. 8, 3, \( \sqrt{73} \)  
9. 8, 10, and 12
7.3 Similar Right Triangles

**MATERIALS** • rectangular piece of paper  • ruler  • scissors  • colored pencils

**QUESTION** How are geometric means related to the altitude of a right triangle?

**EXPLORE** Compare right triangles

**STEP 1**

**Draw a diagonal** Draw a diagonal on your rectangular piece of paper to form two congruent right triangles.

**STEP 2**

**Draw an altitude** Fold the paper to make an altitude to the hypotenuse of one of the triangles.

**STEP 3**

**Cut and label triangles** Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

**STEP 4**

**Arrange the triangles** Arrange the triangles so \( \angle 1, \angle 4, \text{ and } \angle 7 \) are on top of each other as shown.

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. How are the two smaller right triangles related to the large triangle?
2. Explain how you would show that the green triangle is similar to the red triangle.
3. Explain how you would show that the red triangle is similar to the blue triangle.
4. The geometric mean of \( a \) and \( b \) is \( x \) if \( \frac{a}{x} = \frac{x}{b} \). Write a proportion involving the side lengths of two of your triangles so that one side length is the geometric mean of the other two lengths in the proportion.
7.3 Use Similar Right Triangles

Key Vocabulary
- altitude of a triangle, p. 320
- geometric mean, p. 359
- similar polygons, p. 372

Before
You identified the altitudes of a triangle.

Now
You will use properties of the altitude of a right triangle.

Why?
So you can determine the height of a wall, as in Example 4.

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

**THEOREM**

**THEOREM 7.5**

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

\[ \triangle CBD \sim \triangle ABC, \triangle ACD \sim \triangle ABC, \text{ and } \triangle CBD \sim \triangle ACD. \]

*Proof:* below; Ex. 35, p. 456

**Plan for Proof of Theorem 7.5**

First prove that \( \triangle CBS \sim \triangle ABC \). Each triangle has a right angle and each triangle includes \( \angle B \). The triangles are similar by the AA Similarity Postulate. Use similar reasoning to show that \( \triangle ACD \sim \triangle ABC \).

To show \( \triangle CBS \sim \triangle ACD \), begin by showing \( \angle ACD \equiv \angle B \) because they are both complementary to \( \angle DCB \). Each triangle also has a right angle, so you can use the AA Similarity Postulate.

**Example 1** Identify similar triangles

Identify the similar triangles in the diagram.

**Solution**

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

\( \triangle TSU \sim \triangle RTU \sim \triangle RST \)
**Example 2** Find the length of the altitude to the hypotenuse

**SWIMMING POOL** The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?

![Diagram of a swimming pool cross-section](image)

**Solution**

**STEP 1** Identify the similar triangles and sketch them.

![Similar triangles](image)

\[\triangle RST \sim \triangle RTM \sim \triangle TSM\]

**STEP 2** Find the value of \(h\). Use the fact that \(\triangle RST \sim \triangle RTM\) to write a proportion.

\[
\frac{TM}{ST} = \frac{TR}{SR}
\]

Corresponding side lengths of similar triangles are in proportion.

\[
\frac{h}{64} = \frac{152}{165}
\]

Substitute.

\[
165h = 64(152)
\]

Cross Products Property

\[
h = \frac{64(152)}{165}
\]

Solve for \(h\).

\[
h \approx 59
\]

**STEP 3** Read the diagram above. You can see that the maximum depth of the pool is \(h + 48\), which is about 59 + 48 = 107 inches.

The maximum depth of the pool is about 107 inches.

**Guided Practice** for Examples 1 and 2

Identify the similar triangles. Then find the value of \(x\).

1.

![Diagram 1](image)

2.

![Diagram 2](image)
**GEOMETRIC MEANS** In Lesson 6.1, you learned that the geometric mean of two numbers $a$ and $b$ is the positive number $x$ such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $CD$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Notice that $CD$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that $CD$ is the geometric mean of $BD$ and $AD$. As you see below, $CB$ and $AC$ are also geometric means of segment lengths in the diagram.

**Proportions Involving Geometric Means in Right $\triangle ABC$**

- Length of shorter leg of I $\rightarrow \frac{BD}{CD} = \frac{CD}{AD}$ $\rightarrow$ length of longer leg of I
- Length of shorter leg of II $\rightarrow \frac{CD}{AD}$

- Length of hypotenuse of III $\rightarrow \frac{AB}{CB} = \frac{CB}{DB}$ $\rightarrow$ Length of shorter leg of III
- Length of shorter leg of I $\rightarrow \frac{AB}{AC} = \frac{AC}{AD}$ $\rightarrow$ Length of shorter leg of III
- Length of hypotenuse of III $\rightarrow \frac{AB}{AC} = \frac{AC}{AD}$ $\rightarrow$ Length of longer leg of III
- Length of hypotenuse of II $\rightarrow \frac{BD}{CD}$ $\rightarrow$ Length of longer leg of II

**EXAMPLE 3** Use a geometric mean

*Find the value of $y$. Write your answer in simplest radical form.*

**Solution**

**STEP 1** Draw the three similar triangles.

**STEP 2** Write a proportion.

\[
\frac{9}{y} = \frac{y}{3} \quad \text{(Substitute)}
\]

\[
27 = y^2 \quad \text{(Cross Products Property)}
\]

\[
\sqrt{27} = \sqrt{y} \quad \text{(Take the positive square root of each side)}
\]

\[
3\sqrt{3} = y \quad \text{(Simplify)}
\]
THEOREMS

Theorems 7.6 and 7.7 are geometric mean theorems. The first theorem, Geometric Mean (Altitude) Theorem, states that in a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments.

THEOREM 7.6 Geometric Mean (Altitude) Theorem
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

Proof: Ex. 36, p. 456

THEOREM 7.7 Geometric Mean (Leg) Theorem
In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Proof: Ex. 37, p. 456

Example 4: Find a height using indirect measurement
ROCK CLIMBING WALL
To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall.

You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the height of the gym wall.

Solution
By Theorem 7.6, you know that 8.5 is the geometric mean of \( w \) and 5.

\[
\frac{w}{8.5} = \frac{8.5}{5}
\]

Write a proportion.

\[ w \approx 14.5 \]

Solve for \( w \).

So, the height of the wall is \( 5 + w = 5 + 14.5 = 19.5 \) feet.

Guided Practice for Examples 3 and 4
3. In Example 3, which theorem did you use to solve for \( y \)? Explain.
4. Mary is 5.5 feet tall. How far from the wall in Example 4 would she have to stand in order to measure its height?
1. **VOCABULARY** Copy and complete: Two triangles are _?_ if their corresponding angles are congruent and their corresponding side lengths are proportional.

2. **★ WRITING** In your own words, explain geometric mean.

**IDENTIFYING SIMILAR TRIANGLES** Identify the three similar right triangles in the given diagram.

3. 

4. 

**FINDING ALTITUDES** Find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.

5. 

6. 

7. 

**COMPLETING PROPORTIONS** Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.

8. \[
\frac{XW}{?} = \frac{ZW}{YW}
\]

9. \[
\frac{?}{SQ} = \frac{SQ}{TQ}
\]

10. \[
\frac{EF}{EG} = \frac{EG}{?}
\]

**ERROR ANALYSIS** Describe and correct the error in writing a proportion for the given diagram.

11. 

12. 

\[
\frac{w}{z} \neq \frac{z}{w + v}
\]

\[
\frac{e}{d} \neq \frac{d}{f}
\]
**FINDING LENGTHS** Find the value of the variable. Round decimal answers to the nearest tenth.

13. \[
\begin{align*}
x & = 4 \\
5 & \end{align*}
\]

14. \[
\begin{align*}
y & = 18 \\
10 & \end{align*}
\]

15. \[
\begin{align*}
z & = 16 \\
27 & \end{align*}
\]

16. \[
\begin{align*}
x & = 4 \\
9 & \end{align*}
\]

17. \[
\begin{align*}
y & = 8 \\
5 & \end{align*}
\]

18. \[
\begin{align*}
x & = 8 \\
2 & \end{align*}
\]

19. ★ **MULTIPLE CHOICE** Use the diagram at the right. Decide which proportion is false.

- A \[
\frac{DB}{DC} = \frac{DA}{DB}
\]
- B \[
\frac{CA}{AB} = \frac{AB}{AD}
\]
- C \[
\frac{CA}{BA} = \frac{BA}{CA}
\]
- D \[
\frac{DC}{BC} = \frac{BC}{CA}
\]

20. ★ **MULTIPLE CHOICE** In the diagram in Exercise 19 above, \(AC = 36\) and \(BC = 18\). Find \(AD\). If necessary, round to the nearest tenth.

- A 9
- B 15.6
- C 27
- D 31.2

**ALGEBRA** Find the value(s) of the variable(s).

21. \[
\begin{align*}
12 & = a + 5 \\
18 & \end{align*}
\]

22. \[
\begin{align*}
8 & = b + 3 \\
6 & \end{align*}
\]

23. \[
\begin{align*}
y & = 12 \\
16 & \end{align*}
\]

**USING THEOREMS** Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.

24. \[
\begin{align*}
10 & = 8 \\
16 & \end{align*}
\]

25. \[
\begin{align*}
4\sqrt{13} & = 8 \\
12 & \end{align*}
\]

26. \[
\begin{align*}
4\sqrt{33} & = 14 \\
18 & \end{align*}
\]

27. **FINDING LENGTHS** Use the Geometric Mean Theorems to find \(AC\) and \(BD\).

28. **CHALLENGE** Draw a right isosceles triangle and label the two leg lengths \(x\). Then draw the altitude to the hypotome and label its length \(y\). Now draw the three similar triangles and label any side length that is equal to either \(x\) or \(y\). What can you conclude about the relationship between the two smaller triangles? **Explain.**
29. **DOGHOUSE** The peak of the doghouse shown forms a right angle. Use the given dimensions to find the height of the roof.

30. **MONUMENT** You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument. Mary measures the vertical distance from the ground to your eye and the distance from you to the monument. Approximate the height of the monument (as shown at the left below).

31. **SHORT RESPONSE** Paul is standing on the other side of the monument in Exercise 30 (as shown at the right above). He has a piece of rope staked at the base of the monument. He extends the rope to the cardboard square he is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 30? Explain.

32. **PROVING THEOREM 7.1** Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem.

**STATEMENTS**

1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.
2. Draw a perpendicular from $C$ to $AB$.
3. $\frac{c}{a} = \frac{e}{f}$ and $\frac{c}{b} = \frac{f}{e}$
4. $ce = a^2$ and $cf = b^2$
5. $ce + f^2 = \left( \frac{c}{a} \right)^2 + f^2$
6. $ce + cf = a^2 + b^2$
7. $c(e + f) = a^2 + b^2$
8. $e + f = \_\_\_?
9. $c \cdot c = a^2 + b^2$
10. $c^2 = a^2 + b^2$

**REASONS**

1. ?
2. Perpendicular Postulate
3. ?
4. ?
5. Addition Property of Equality
6. ?
7. ?
8. Segment Addition Postulate
9. ?
10. Simplify.
33. **MULTI-STEP PROBLEM** Use the diagram.
   a. Name all the altitudes in \( \triangle EGF \). *Explain.*
   b. Find \( FH \).
   c. Find the area of the triangle.

34. **★ EXTENDED RESPONSE** Use the diagram.
   a. Sketch the three similar triangles in the diagram. Label the vertices. *Explain* how you know which vertices correspond.
   b. Write similarity statements for the three triangles.
   c. Which segment’s length is the geometric mean of RT and RQ? *Explain* your reasoning.

**PROVING THEOREMS** In Exercises 35–37, use the diagram and GIVEN statements below.

**GIVEN** \( \triangle ABC \) is a right triangle.
Altitude \( CD \) is drawn to hypotenuse \( AB \).

35. Prove Theorem 7.5 by using the Plan for Proof on page 449.
36. Prove Theorem 7.6 by showing \( \frac{BD}{CD} = \frac{CD}{AD} \).
37. Prove Theorem 7.7 by showing \( \frac{AB}{CB} = \frac{CB}{DB} \) and \( \frac{AB}{AC} = \frac{AC}{AD} \).

38. **CHALLENGE** The harmonic mean of \( a \) and \( b \) is \( \frac{2ab}{a + b} \). The Greek mathematician Pythagoras found that three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the harmonic mean of the lengths of the shortest and longest string.
   a. Find the harmonic mean of 10 and 15.
   b. Find the harmonic mean of 6 and 14.
   c. Will equally taut strings whose lengths have the ratio 4 : 6 : 12 sound harmonious? *Explain* your reasoning.

---

**MIXED REVIEW**

**PREVIEW** Prepare for Lesson 7.4 in Exs. 39–46.

<table>
<thead>
<tr>
<th>Simplify the expression. (p. 874)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{27} \cdot \sqrt{2} )</td>
</tr>
<tr>
<td>( \sqrt{8} \cdot \sqrt{10} )</td>
</tr>
<tr>
<td>( \sqrt{12} \cdot \sqrt{7} )</td>
</tr>
<tr>
<td>( \sqrt{18} \cdot \sqrt{12} )</td>
</tr>
<tr>
<td>( \frac{5}{\sqrt{7}} )</td>
</tr>
<tr>
<td>( \frac{8}{\sqrt{11}} )</td>
</tr>
<tr>
<td>( \frac{15}{\sqrt{27}} )</td>
</tr>
<tr>
<td>( \frac{12}{\sqrt{24}} )</td>
</tr>
</tbody>
</table>

Tell whether the lines through the given points are **parallel**, **perpendicular**, or **neither**. *Justify* your answer. (p. 171)

<table>
<thead>
<tr>
<th>Line 1: ((2, 4), (4, 2))</th>
<th>Line 1: ((0, 2), (-1, -1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 2: ((3, 5), (-1, 1))</td>
<td>Line 2: ((3, 1), (1, -5))</td>
</tr>
<tr>
<td>Line 2: ((5, 2), (7, 4))</td>
<td></td>
</tr>
</tbody>
</table>
7.4 Special Right Triangles

You found side lengths using the Pythagorean Theorem.

You will use the relationships among the sides in special right triangles.

So you can find the height of a drawbridge, as in Ex. 28.

**Key Vocabulary**

- isosceles triangle, p. 217

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half as shown.

**Theorem**

**Theorem 7.8** **45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

\[
\text{hypotenuse} = \text{leg} \cdot \sqrt{2}
\]

*Proof:* Ex. 30, p. 463

**Example 1** Find hypotenuse length in a 45°-45°-90° triangle

Find the length of the hypotenuse.

a. 

\[
\begin{align*}
\text{hypotenuse} & = \text{leg} \cdot \sqrt{2} \\
& = 8 \sqrt{2} \\
& = 8 \cdot \sqrt{2} \\
& = 8 \cdot 1.41 \\
& = 11.3
\end{align*}
\]

b. 

\[
\begin{align*}
\text{hypotenuse} & = \text{leg} \cdot \sqrt{2} \\
& = 3 \sqrt{2} \cdot \sqrt{2} \\
& = 3 \cdot 2 \\
& = 6
\end{align*}
\]

**Review Algebra**

Remember the following properties of radicals:

\[
\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}
\]

\[
\sqrt{a} \cdot a = a
\]

For a review of radical expressions, see p. 874.
EXAMPLE 2  Find leg lengths in a $45^\circ-45^\circ-90^\circ$ triangle

Find the lengths of the legs in the triangle.

Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a $45^\circ-45^\circ-90^\circ$ triangle.

\[
\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \quad \text{45°-45°-90° Triangle Theorem}
\]
\[
5\sqrt{2} = x \cdot \sqrt{2} \quad \text{Substitute.}
\]
\[
\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}} \quad \text{Divide each side by} \ \sqrt{2}.
\]
\[
5 = x \quad \text{Simplify.}
\]

EXAMPLE 3  Standardized Test Practice

Triangle $WXY$ is a right triangle.
Find the length of $WX$.

\begin{align*}
\text{A} & \quad 50 \text{ cm} \\
\text{B} & \quad 25\sqrt{2} \text{ cm} \\
\text{C} & \quad 25 \text{ cm} \\
\text{D} & \quad \frac{25\sqrt{2}}{2} \text{ cm}
\end{align*}

Solution

By the Corollary to the Triangle Sum Theorem, the triangle is a $45^\circ-45^\circ-90^\circ$ triangle.

\[
\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \quad \text{45°-45°-90° Triangle Theorem}
\]
\[
WX = 25\sqrt{2} \quad \text{Substitute.}
\]

\>

The correct answer is B.  \(\text{A} \quad \text{B} \quad \text{C} \quad \text{D}\)

GUIDED PRACTICE  for Examples 1, 2, and 3

Find the value of the variable.

1. \[
\begin{align*}
2\sqrt{2} & \quad x
\end{align*}
\]

2. \[
\begin{align*}
\sqrt{2} & \quad y \quad \sqrt{2}
\end{align*}
\]

3. \[
\begin{align*}
8 & \quad 0 \quad 8
\end{align*}
\]

4. Find the leg length of a $45^\circ-45^\circ-90^\circ$ triangle with a hypotenuse length of 6.
A 30°-60°-90° triangle can be formed by dividing an equilateral triangle in half.

**Theorem 7.9** 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

\[
\text{hypotenuse} = 2 \cdot \text{shorter leg}
\]
\[
\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}
\]

**Proof:** Ex. 32, p. 463

---

**Example 5** Find lengths in a 30°-60°-90° triangle

**Solution**

Find the values of $x$ and $y$. Write your answer in simplest radical form.

**Step 1** Find the value of $x$.

longer leg = shorter leg $\cdot \sqrt{3}$

\[
9 = x\sqrt{3}
\]

\[
\frac{9}{\sqrt{3}} = x
\]

\[
\frac{9\sqrt{3}}{\sqrt{3}} = x
\]

\[
\frac{9\sqrt{3}}{3} = x
\]

\[
3\sqrt{3} = x
\]

**Step 2** Find the value of $y$.

hypotenuse = 2 $\cdot$ shorter leg

\[
y = 2 \cdot 3\sqrt{3} = 6\sqrt{3}
\]
EXAMPLE 6  Find a height

**DUMP TRUCK** The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

a. 45° angle  

**Solution**

a. When the body is raised 45° above the frame, the height \( h \) is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 14 feet.

\[
14 = h \cdot \sqrt{2} \quad 45^\circ-45^\circ-90^\circ \text{ Triangle Theorem}
\]

\[
\frac{14}{\sqrt{2}} = h \quad \text{Divide each side by } \sqrt{2}.
\]

\[
9.9 \approx h \quad \text{Use a calculator to approximate.}
\]

When the angle of elevation is 45°, the body is about 9 feet 11 inches above the frame.

b. When the body is raised 60°, the height \( h \) is the length of the longer leg of a 30°-60°-90° triangle. The length of the hypotenuse is 14 feet.

\[
\text{hypotenuse} = 2 \cdot \text{shorter leg} \quad 30^\circ-60^\circ-90^\circ \text{ Triangle Theorem}
\]

\[
14 = 2 \cdot s \quad \text{Substitute.}
\]

\[
7 = s \quad \text{Divide each side by 2.}
\]

\[
\text{longer leg} = \text{shorter leg} \cdot \sqrt{3} \quad 30^\circ-60^\circ-90^\circ \text{ Triangle Theorem}
\]

\[
h = 7\sqrt{3} \quad \text{Substitute.}
\]

\[
h \approx 12.1 \quad \text{Use a calculator to approximate.}
\]

When the angle of elevation is 60°, the body is about 12 feet 1 inch above the frame.

---

**GUIDED PRACTICE** for Examples 4, 5, and 6

Find the value of the variable.

5. \[
\sqrt{3} \quad 30^\circ \quad x
\]

6. \[
\begin{array}{c}
4 \\
\hline
2 \\
\hline
2
\end{array}
\]

7. **WHAT IF?** In Example 6, what is the height of the body of the dump truck if it is raised 30° above the frame?

8. In a 30°-60°-90° triangle, describe the location of the shorter side. Describe the location of the longer side?
1. **VOCABULARY** Copy and complete: A triangle with two congruent sides and a right angle is called ?.

2. ★ **WRITING** Explain why the acute angles in an isosceles right triangle always measure 45°.

**45°-45°-90° TRIANGLES** Find the value of \( x \). Write your answer in simplest radical form.

3. 
4. 
5. 

**MULTIPLE CHOICE** Find the length of \( \overline{AC} \).

\[ \begin{align*}
\text{A} & : 7\sqrt{2} \text{ in.} \\
\text{B} & : 2\sqrt{7} \text{ in.} \\
\text{C} & : \frac{7\sqrt{2}}{2} \text{ in.} \\
\text{D} & : \sqrt{14} \text{ in.}
\end{align*} \]

**ISOSCELES RIGHT TRIANGLE** The square tile shown has painted corners in the shape of congruent 45°-45°-90° triangles. What is the value of \( x \)? What is the side length of the tile?

**30°-60°-90° TRIANGLES** Find the value of each variable. Write your answers in simplest radical form.

6. 
7.
8. 
9. 
10.

**SPECIAL RIGHT TRIANGLES** Copy and complete the table.

\[ \begin{array}{c|c|c|c|c|c}
\hline
& a & b & c & d & e \\
\hline
\text{11.} & 7 & ? & \sqrt{5} & 5 & ? \\
\text{12.} & ? & 11 & ? & ? & 8\sqrt{3} \\
\text{c} & ? & 10 & 6\sqrt{2} & ? & ? \\
\text{f} & ? & 14 & ? & 18\sqrt{3} & ? \\
\hline
\end{array} \]
**ALGEBRA** Find the value of each variable. Write your answers in simplest radical form.

13. $\sqrt{30}$
14. $\sqrt{6}$
15. $\sqrt{5}$
16. $\sqrt{2}$
17. $\sqrt{3}$
18. $\sqrt{3}$

19. ★ **MULTIPLE CHOICE** Which side lengths do not represent a 30°-60°-90° triangle?

- (A) $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$
- (B) $\sqrt{2}, \sqrt{6}, 2\sqrt{2}$
- (C) $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$
- (D) $3, 3\sqrt{3}, 6$

**ERROR ANALYSIS** Describe and correct the error in finding the length of the hypotenuse.

20. $7\sqrt{3}$
21. $5\sqrt{2}$

22. ★ **WRITING** Abigail solved Example 5 on page 459 in a different way. Instead of dividing each side by $\sqrt{3}$, she multiplied each side by $\sqrt{3}$. Does her method work? Explain why or why not.

**ALGEBRA** Find the value of each variable. Write your answers in simplest radical form.

23. $\sqrt{10}$
24. $\sqrt{4}$
25. $\sqrt{8}$

26. **CHALLENGE** $\triangle ABC$ is a 30°-60°-90° triangle. Find the coordinates of $A$.  

★ = STANDARDIZED TEST PRACTICE
27. **KAYAK RAMP** A ramp is used to launch a kayak. What is the height of an 11 foot ramp when its angle is 30° as shown?

28. **DRAWBRIDGE** Each half of the drawbridge is about 284 feet long, as shown. How high does a seagull rise who is on the end of the drawbridge when the angle with measure \( x \) is 30°, 45°, 60°?

29. **SHORT RESPONSE** Describe two ways to show that all isosceles right triangles are similar to each other.

30. **PROVING THEOREM 7.8** Write a paragraph proof of the 45°-45°-90° Triangle Theorem.
   **GIVEN** △DEF is a 45°-45°-90° triangle.
   **PROVE** The hypotenuse is \( \sqrt{2} \) times as long as each leg.

31. **EQUILATERAL TRIANGLE** If an equilateral triangle has a side length of 20 inches, find the height of the triangle.

32. **PROVING THEOREM 7.9** Write a paragraph proof of the 30°-60°-90° Triangle Theorem.
   **GIVEN** △JKL is a 30°-60°-90° triangle.
   **PROVE** The hypotenuse is twice as long as the shorter leg and the longer leg is \( \sqrt{3} \) times as long as the shorter leg.

   **Plan for Proof** Construct △JML congruent to △JKL. Then prove that △JLM is equilateral. Express the lengths of \( JK \) and \( JL \) in terms of \( x \).

33. **MULTI-STEP PROBLEM** You are creating a quilt that will have a traditional “flying geese” border, as shown below.
   a. Find all the angle measures of the small blue triangles and the large orange triangles.
   b. The width of the border is to be 3 inches. To create the large triangle, you cut a square of fabric in half. Not counting any extra fabric needed for seams, what size square do you need?
   c. What size square do you need to create each small triangle?
34. **EXTENDED RESPONSE** Use the figure at the right. You can use the fact that the converses of the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem are true.

a. Find the values of $r$, $s$, $t$, $u$, $v$, and $w$. Explain the procedure you used to find the values.

b. Which of the triangles, if any, is a 45°-45°-90° triangle? Explain.

c. Which of the triangles, if any, is a 30°-60°-90° triangle? Explain.

35. **CHALLENGE** In quadrilateral $QRST$, $m\angle R = 60^\circ$, $m\angle T = 90^\circ$, $QR = RS$, $ST = 8$, $TQ = 8$, and $\overline{RT}$ and $\overline{QS}$ intersect at point $Z$.

a. Draw a diagram.

b. Explain why $\triangle RQT \cong \triangle RST$.

c. Which is longer, $QS$ or $RT$? Explain.

---

**MIXED REVIEW**

In the diagram, $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. (p. 303)

36. Which pairs of segment lengths are equal?

37. What is the value of $x$?

38. Find $CD$.

Is it possible to build a triangle using the given side lengths? (p. 328)

39. 4, 4, and 7

40. 3, 3, and $9\sqrt{2}$

41. 7, 15, and 21

Tell whether the given side lengths form a right triangle. (p. 441)

42. 21, 22, and $5\sqrt{37}$

43. $\frac{3}{2}$, 2, and $\frac{5}{2}$

44. 8, 10, and 14

---

**QUIZ for Lessons 7.3–7.4**

In Exercises 1 and 2, use the diagram. (p. 449)

1. Which segment’s length is the geometric mean of $\overline{AC}$ and $\overline{CD}$?

2. Find $BD$, $AD$, and $AB$.

Find the values of the variable(s). Write your answer(s) in simplest radical form. (p. 457)

3. $x$

4. $y$

5. $a$
Lessons 7.1–7.4

1. **GRIDDED ANSWER** Find the direct distance, in paces, from the treasure to the stump.

```
From the old stump,
take 30 paces east,
then 20 paces north,
6 paces west, and then
another 25 paces north
to find the hidden treasure.
```

2. **MULTI-STEP PROBLEM** On a map of the United States, you put a pushpin on three state capitols you want to visit: Jefferson City, Missouri; Little Rock, Arkansas; and Atlanta, Georgia.

   a. Draw a diagram to model the triangle.
   b. Do the pushpins form a right triangle? If not, what type of triangle do they form?

3. **SHORT RESPONSE** Bob and John started running at 10 A.M. Bob ran east at 4 miles per hour while John ran south at 5 miles per hour. How far apart were they at 11:30 A.M.? Describe how you calculated the answer.

4. **EXTENDED RESPONSE** Give all values of \( x \) that make the statement true for the given diagram.

```
\[ \begin{array}{c}
8 \\
 \hline
 x \\
 \hline
6
\end{array} \]
```

   a. \( \angle 1 \) is a right angle. *Explain.*
   b. \( \angle 1 \) is an obtuse angle. *Explain.*
   c. \( \angle 1 \) is an acute angle. *Explain.*
   d. The triangle is isosceles. *Explain.*
   e. No triangle is possible. *Explain.*

5. **EXTENDED RESPONSE** A Chinese checker board is made of triangles. Use the picture below to answer the questions.

```
A Chinese checker board is made of triangles. Use the picture below to answer the questions.
```

   a. Count the marble holes in the purple triangle. What kind of triangle is it?
   b. If a side of the purple triangle measures 8 centimeters, find the area of the purple triangle.
   c. How many marble holes are in the center hexagon? Assuming each marble hole takes up the same amount of space, what is the relationship between the purple triangle and center hexagon?
   d. Find the area of the center hexagon. *Explain* your reasoning.

6. **MULTI-STEP PROBLEM** You build a beanbag toss game. The game is constructed from a sheet of plywood supported by two boards. The two boards form a right angle and their lengths are 3 feet and 2 feet.

```
\[ \begin{array}{c}
3 \text{ ft} \\
 \hline
 x \\
 \hline
2 \text{ ft}
\end{array} \]
```

   a. Find the length \( x \) of the plywood.
   b. You put in a support that is the altitude \( y \) to the hypotenuse of the right triangle. What is the length of the support?
   c. Where does the support attach to the plywood? *Explain.*
### Key Vocabulary

- trigonometric ratio
- tangent

### Activity Right Triangle Ratio

**Materials:** metric ruler, protractor, calculator

**STEP 1** Draw a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

**STEP 2** Measure the legs of each right triangle. Copy and complete the table.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Adjacent leg</th>
<th>Opposite leg</th>
<th>Opposite leg Adjacent leg</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ABC</td>
<td>5 cm</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>△ADE</td>
<td>10 cm</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>△AFG</td>
<td>15 cm</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**STEP 3** Explain why the proportions \(\frac{BC}{DE} = \frac{AC}{AE}\) and \(\frac{BC}{AC} = \frac{DE}{AE}\) are true.

**STEP 4** Make a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.

The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

### Key Concept

**Tangent Ratio**

Let \(\triangle ABC\) be a right triangle with acute \(\angle A\).

The tangent of \(\angle A\) (written as \(\tan A\)) is defined as follows:

\[
\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}
\]

**Abbreviate**

Remember these abbreviations:
- tangent → tan
- opposite → opp.
- adjacent → adj.
**COMPLEMENTARY ANGLES** In the right triangle, $\angle A$ and $\angle B$ are complementary so you can use the same diagram to find the tangent of $\angle A$ and the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg opposite $\angle B$ and the leg opposite $\angle A$ is the leg adjacent to $\angle B$.

**EXAMPLE 1** Find tangent ratios

Find $\tan S$ and $\tan R$. Write each answer as a fraction and as a decimal rounded to four places.

**Solution**

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

**GUIDED PRACTICE** for Example 1

Find $\tan J$ and $\tan K$. Round to four decimal places.

1. $J$

2. $L$

**EXAMPLE 2** Find a leg length

**ALGEBRA** Find the value of $x$.

**Solution**

Use the tangent of an acute angle to find a leg length.

$$\tan 32° = \frac{\text{opp.}}{\text{adj.}}$$

Write ratio for tangent of 32°.

$$\tan 32° = \frac{11}{x}$$

Substitute.

$$x \cdot \tan 32° = 11$$

Multiply each side by $x$.

$$x = \frac{11}{\tan 32°}$$

Divide each side by $\tan 32°$.

$$x \approx \frac{11}{0.6249}$$

Use a calculator to find $\tan 32°$.

$$x \approx 17.6$$

Simplify.
**Example 3**  \textbf{Estimate height using tangent}

**LAMPPOST** Find the height $h$ of the lamppost to the nearest inch.

\[
\tan 70^\circ = \frac{\text{opp.}}{\text{adj.}} \quad \text{Write ratio for tangent of 70}.^\circ
\]

\[
\tan 70^\circ = \frac{h}{40} \quad \text{Substitute.}
\]

\[
40 \cdot \tan 70^\circ = h \quad \text{Multiply each side by 40.}
\]

\[
109.9 \approx h \quad \text{Use a calculator to simplify.}
\]

The lamppost is about 110 inches tall.

**Special Right Triangles** You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

**Example 4**  \textbf{Use a special right triangle to find a tangent}

Use a special right triangle to find the tangent of a 60° angle.

**Step 1** Because all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.

\[
\text{longer leg} = \text{shorter leg} \cdot \sqrt{3} \quad 30^\circ-60^\circ-90^\circ \text{ Triangle Theorem}
\]

\[
x = 1 \cdot \sqrt{3} \quad \text{Substitute.}
\]

\[
x = \sqrt{3} \quad \text{Simplify.}
\]

**Step 2** Find $\tan 60^\circ$.

\[
\tan 60^\circ = \frac{\text{opp.}}{\text{adj.}} \quad \text{Write ratio for tangent of 60}.^\circ
\]

\[
\tan 60^\circ = \frac{\sqrt{3}}{1} \quad \text{Substitute.}
\]

\[
\tan 60^\circ = \sqrt{3} \quad \text{Simplify.}
\]

The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

**Guided Practice** for Examples 2, 3, and 4

Find the value of $x$. Round to the nearest tenth.

3. \[
\begin{align*}
x & = 61^\circ \\
& \quad 22
\end{align*}
\]

4. \[
\begin{align*}
x & = 13 \\
& \quad 56
\end{align*}
\]

5. **What if?** In Example 4, suppose the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$. 

468  Chapter 7  Right Triangles and Trigonometry
1. **VOCABULARY** Copy and complete: The tangent ratio compares the length of ____ to the length of ____.

2. ★ **WRITING** Explain how you know that all right triangles with an acute angle measuring $n^\circ$ are similar to each other.

**FINDING TANGENT RATIOS** Find $\tan A$ and $\tan B$. Write each answer as a fraction and as a decimal rounded to four places.

3. \[ \begin{array}{c}
\triangle ABC \\
A \quad 25 \\
B \quad 7 \\
C \quad 24 \\
\end{array} \]

4. \[ \begin{array}{c}
\triangle ABC \\
A \quad 35 \\
B \quad 12 \\
C \quad 37 \\
\end{array} \]

5. \[ \begin{array}{c}
\triangle ABC \\
A \quad 52 \\
B \quad 20 \\
C \quad 48 \\
\end{array} \]

**FINDING LEG LENGTHS** Find the value of $x$ to the nearest tenth.

6. \[ \begin{array}{c}
\triangle \quad 12 \\
\quad 41 \degree \\
x \quad 18 \\
\end{array} \]

7. \[ \begin{array}{c}
\triangle \quad 15 \\
\quad 27 \degree \\
x \quad 82 \\
\end{array} \]

8. \[ \begin{array}{c}
\triangle \quad 22 \\
\quad 58 \degree \\
x \quad 80 \\
\end{array} \]

**FINDING LEG LENGTHS** Find the value of $x$ using the definition of tangent. Then find the value of $x$ using the $45^\circ$-$45^\circ$-$90^\circ$ Theorem or the $30^\circ$-$60^\circ$-$90^\circ$ Theorem. Compare the results.

9. \[ \begin{array}{c}
\triangle \quad 6 \\
\quad 45 \degree \\
x \quad 8 \sqrt{3} \\
\end{array} \]

10. \[ \begin{array}{c}
\triangle \quad 10 \sqrt{3} \\
\quad 30 \degree \\
x \quad 4 \sqrt{3} \\
\end{array} \]

11. \[ \begin{array}{c}
\triangle \quad 60 \degree \\
\quad 4 \\
x \quad 8 \sqrt{3} \\
\end{array} \]

12. **SPECIAL RIGHT TRIANGLES** Find $\tan 30^\circ$ and $\tan 45^\circ$ using the $45^\circ$-$45^\circ$-$90^\circ$ Triangle Theorem and the $30^\circ$-$60^\circ$-$90^\circ$ Triangle Theorem.

**ERROR ANALYSIS** Describe the error in the statement of the tangent ratio. Correct the statement, if possible. Otherwise, write not possible.

13. \[ \tan D = \frac{18}{82} \]

14. \[ \tan 55^\circ = \frac{18}{BC} \]

15. ★ **WRITING** Describe what you must know about a triangle in order to use the tangent ratio.
16. ★ MULTIPLE CHOICE  Which expression can be used to find the value of $x$ in the triangle shown?

- **A** $x = 20 \cdot \tan 40^\circ$
- **B** $x = \frac{\tan 40^\circ}{20}$
- **C** $x = \frac{20}{\tan 40^\circ}$
- **D** $x = \frac{20}{\tan 50^\circ}$

17. ★ MULTIPLE CHOICE  What is the approximate value of $x$ in the triangle shown?

- **A** 0.4
- **B** 2.7
- **C** 7.5
- **D** 19.2

FINDING LEG LENGTHS  Use a tangent ratio to find the value of $x$. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.

18.  

19.  

20.  

FINDING AREA  Find the area of the triangle. Round to the nearest tenth.

21.  

22.  

23.  

FINDING PERIMETER  Find the perimeter of the triangle. Round to the nearest tenth.

24.  

25.  

26.  

FINDING LENGTHS  Find $y$. Then find $z$. Round to the nearest tenth.

27.  

28.  

29.  

30. CHALLENGE  Find the perimeter of the figure at the right, where $AC = 26$, $AD = BF$, and $D$ is the midpoint of $AC$.

WORKED-OUT SOLUTIONS on p. WS1  ★ STANDARDIZED TEST PRACTICE
31. **WASHINGTON MONUMENT** A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle between the ground and the top of the monument to be 78°. Find the height \( h \) of the Washington Monument to the nearest foot.

32. **ROLLER COASTERS** A roller coaster makes an angle of 52° with the ground. The horizontal distance from the crest of the hill to the bottom of the hill is about 121 feet, as shown. Find the height \( h \) of the roller coaster to the nearest foot.

**CLASS PICTURE** Use this information and diagram for Exercises 33 and 34.

Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. If she looks toward either end of the class, she turns 50°.

**33. ISOSCELES TRIANGLE** What is the distance between the ends of the class?

**34. MULTI-STEP PROBLEM** The photographer wants to estimate how many more students can fit at the end of the first row. The photographer turns 50° to see the last student and another 10° to see the end of the camera range.

a. Find the distance from the center to the last student in the row.

b. Find the distance from the center to the end of the camera range.

c. Use the results of parts (a) and (b) to estimate the length of the empty space.

d. If each student needs 2 feet of space, about how many more students can fit at the end of the first row? *Explain* your reasoning.

**35.★ SHORT RESPONSE** Write expressions for the tangent of each acute angle in the triangle. *Explain* how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair are \( \angle A \) and \( \angle B \)?
36. **EYE CHART** You are looking at an eye chart that is 20 feet away. Your eyes are level with the bottom of the “E” on the chart. To see the top of the “E,” you look up 1°. How tall is the “E”?

37. **★ EXTENDED RESPONSE** According to the Americans with Disabilities Act, a ramp cannot have an incline that is greater than 5°. The regulations also state that the maximum rise of a ramp is 30 inches. When a ramp needs to reach a height greater than 30 inches, a series of ramps connected by 60 inch landings can be used, as shown below.

   ![Ramp Diagram](image)

   a. What is the maximum horizontal length of the base of one ramp, in feet? Round to the nearest foot.

   b. If a doorway is 7.5 feet above the ground, what is the least number of ramps and landings you will need to lead to the doorway? Draw and label a diagram to justify your answer.

   c. To the nearest foot, what is the total length of the base of the system of ramps and landings in part (b)?

38. **CHALLENGE** The road salt shown is stored in a cone-shaped pile. The base of the cone has a circumference of 80 feet. The cone rises at an angle of 32°. Find the height \( h \) of the cone. Then find the length \( s \) of the cone-shaped pile.

---

**MIXED REVIEW**

The expressions given represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. (p. 217)

39. \( m \angle A = x^\circ \) \n   \( m \angle B = 4x^\circ \) \n   \( m \angle C = 4x^\circ \)

40. \( m \angle A = x^\circ \) \n   \( m \angle B = x^\circ \) \n   \( m \angle C = (5x - 60)^\circ \)

41. \( m \angle A = (x + 20)^\circ \) \n   \( m \angle B = (3x + 15)^\circ \) \n   \( m \angle C = (x - 30)^\circ \)

Copy and complete the statement with <, >, or =. Explain. (p. 335)

42. \( m \angle 1 \ ? \ m \angle 2 \)

43. \( m \angle 1 \ ? \ m \angle 2 \)

44. \( m \angle 1 \ ? \ m \angle 2 \)

Find the unknown side length of the right triangle. (p. 433)

45. \( 18 \)

46. \( x \)

47. \( x \)
7.6 Apply the Sine and Cosine Ratios

**Before**
You used the tangent ratio.

**Now**
You will use the sine and cosine ratios.

**Why**
So you can find distances, as in Ex. 39.

**Key Vocabulary**
- sine
- cosine
- angle of elevation
- angle of depression

The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

**Abbreviate**
Remember these abbreviations:
- sine → sin
- cosine → cos
- hypotenuse → hyp

**Key Concept**

**Sine and Cosine Ratios**

Let \( \triangle ABC \) be a right triangle with acute \( \angle A \).

The sine of \( \angle A \) and cosine of \( \angle A \) (written \( \sin A \) and \( \cos A \)) are defined as follows:

\[
\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}
\]

\[
\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}
\]

**Example 1**

**Find sine ratios**

Find \( \sin S \) and \( \sin R \). Write each answer as a fraction and as a decimal rounded to four places.

**Solution**

\[
\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} = 0.9692
\]

\[
\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} = 0.2462
\]

**Guided Practice** for Example 1

Find \( \sin X \) and \( \sin Y \). Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1. 
   \[
   \begin{array}{c}
   X \\
   17 \\
   \hline
   15 \\
   Z \\
   \hline
   Y \\
   \end{array}
   \]

2. 
   \[
   \begin{array}{c}
   X \\
   20 \\
   \hline
   25 \\
   Z \\
   \hline
   Y \\
   \end{array}
   \]
**Example 2**  
Find cosine ratios

Find \( \cos U \) and \( \cos W \). Write each answer as a fraction and as a decimal.

**Solution**

\[
\cos U = \frac{\text{adj. to } \angle U}{\text{hyp}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000
\]

\[
\cos W = \frac{\text{adj. to } \angle W}{\text{hyp}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000
\]

**Example 3**  
Use a trigonometric ratio to find a hypotenuse

**DOG RUN** You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.

**Solution**

\[
\sin 35^\circ = \frac{\text{opp.}}{\text{hyp.}}
\]

\[
\sin 35^\circ = \frac{11}{x}
\]

Multiply each side by \( x \).

\[
x \cdot \sin 35^\circ = 11
\]

Divide each side by \( \sin 35^\circ \).

\[
x = \frac{11}{\sin 35^\circ}
\]

Use a calculator to find \( \sin 35^\circ \).

\[
x = \frac{11}{0.5736}
\]

Simplify.

\[
x = 19.2
\]

You will need a little more than 19 feet of cable.

**Guided Practice**  
for Examples 2 and 3

In Exercises 3 and 4, find \( \cos R \) and \( \cos S \). Write each answer as a decimal. Round to four decimal places, if necessary.

3. \( \begin{array}{c} T \\ 9 \end{array} \quad \begin{array}{c} 12 \quad S \\ R \end{array} \)

4. \( \begin{array}{c} S \\ 16 \end{array} \quad \begin{array}{c} R \\ 30 \end{array} \)

5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed.
**Example 4** Find a hypotenuse using an angle of depression

**Skiing** You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?

**Solution**

\[
\sin 21^\circ = \frac{\text{opp}}{\text{hyp}}
\]

Write ratio for sine of 21°.

\[
\sin 21^\circ = \frac{1200}{x}
\]

Substitute.

\[
x \cdot \sin 21^\circ = 1200
\]

Multiply each side by \(x\).

\[
x = \frac{1200}{\sin 21^\circ}
\]

Divide each side by \(\sin 21^\circ\).

\[
x \approx \frac{1200}{0.3584}
\]

Use a calculator to find \(\sin 21^\circ\).

\[
x \approx 3348.2
\]

Simplify.

You ski about 3348 meters down the mountain.

**Guided Practice**

6. **What if?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski?
**Example 5** Find leg lengths using an angle of elevation

**SKATEBOARD RAMP** You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of $26^\circ$. You need to find the height and length of the base of the ramp.

**Solution**

**STEP 1** Find the height.

$$\sin 26^\circ = \frac{\text{opp.}}{\text{hyp.}}$$  \[\text{Write ratio for sine of } 26^\circ.\]

$$\sin 26^\circ = \frac{x}{14}$$  \[\text{Substitute.}\]

$$14 \cdot \sin 26^\circ = x$$  \[\text{Multiply each side by 14.}\]

$$6.1 \approx x$$  \[\text{Use a calculator to simplify.}\]

$\rightarrow$ The height is about 6.1 feet.

**STEP 2** Find the length of the base.

$$\cos 26^\circ = \frac{\text{adj.}}{\text{hyp.}}$$  \[\text{Write ratio for cosine of } 26^\circ.\]

$$\cos 26^\circ = \frac{y}{14}$$  \[\text{Substitute.}\]

$$14 \cdot \cos 26^\circ = y$$  \[\text{Multiply each side by 14.}\]

$$12.6 \approx y$$  \[\text{Use a calculator to simplify.}\]

$\rightarrow$ The length of the base is about 12.6 feet.

**Example 6** Use a special right triangle to find a sine and cosine

Use a special right triangle to find the sine and cosine of a $60^\circ$ angle.

**Solution**

Use the $30^\circ$-$60^\circ$-$90^\circ$ Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the $60^\circ$ angle.

$$\sin 60^\circ = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60^\circ = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$

**Guided Practice** for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is $35^\circ$. What is the new height and base length of the ramp?

8. Use a special right triangle to find the sine and cosine of a $30^\circ$ angle.
1. **VOCABULARY** Copy and complete: The sine ratio compares the length of _?_ to the length of _?_.

2. **WRITING** Explain how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

**FINDING SINE RATIOS** Find \( \sin D \) and \( \sin E \). Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

3. \[
\begin{align*}
\sin D &= \frac{9}{15} \\
\sin E &= \frac{12}{15}
\end{align*}
\]

4. \[
\begin{align*}
\sin D &= \frac{37}{35} \\
\sin E &= \frac{12}{53}
\end{align*}
\]

5. \[
\sin D = \frac{45}{53} \\
\sin F = \frac{28}{53}
\]

6. **ERROR ANALYSIS** Explain why the student’s statement is incorrect. Write a correct statement for the sine of the angle.

**FINDING COSINE RATIOS** Find \( \cos X \) and \( \cos Y \). Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

7. \[
\begin{align*}
\cos X &= 45 \\
\cos Y &= 36
\end{align*}
\]

8. \[
\begin{align*}
\cos X &= 15 \\
\cos Z &= 17
\end{align*}
\]

9. \[
\begin{align*}
\cos X &= 26 \\
\cos Y &= 13 \sqrt{3}
\end{align*}
\]

**USING SINE AND COSINE RATIOS** Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.

10. \[
\begin{align*}
x &= 18 \\
y &= 32°
\end{align*}
\]

11. \[
\begin{align*}
b &= 10 \\
g &= 48°
\end{align*}
\]

12. \[
\begin{align*}
w &= 10° \\
v &= 5
\end{align*}
\]

13. \[
\begin{align*}
s &= 26 \\
r &= 43°
\end{align*}
\]

14. \[
\begin{align*}
p &= 34 \\
q &= 64°
\end{align*}
\]

15. \[
\begin{align*}
m &= 8 \\
n &= 50°
\end{align*}
\]

16. **SPECIAL RIGHT TRIANGLES** Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle.
17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio.

18. ★ **MULTIPLE CHOICE** In \( \triangle PQR \), which expression can be used to find \( PQ \)?

   - A) \( 10 \cdot \cos 29^\circ \)
   - B) \( 10 \cdot \sin 29^\circ \)
   - C) \( \frac{10}{\sin 29^\circ} \)
   - D) \( \frac{10}{\cos 29^\circ} \)

19. **ALGEBRA** Find the value of \( x \). Round decimals to the nearest tenth.

20. **FINDING SINE AND COSINE RATIOS** Find the unknown side length. Then find \( \sin X \) and \( \cos X \). Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.

21. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle.

22. ★ **MULTIPLE CHOICE** In \( \triangle JKL \), \( m \angle L = 90^\circ \). Which statement about \( \triangle JKL \) cannot be true?

   - A) \( \sin J = 0.5 \)
   - B) \( \sin J = 0.1071 \)
   - C) \( \sin J = 0.8660 \)
   - D) \( \sin J = 1.1 \)

23. **PERIMETER** Find the approximate perimeter of the figure.

24. **CHALLENGE** Let \( A \) be any acute angle of a right triangle. Show that

   (a) \( \tan A = \frac{\sin A}{\cos A} \) and (b) \( (\sin A)^2 + (\cos A)^2 = 1 \).
33. AIRPLANE RAMP The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length \( y \) of the ramp?

34. BLEACHERS Find the horizontal distance \( h \) the bleachers cover. Round to the nearest foot.

35. ★ SHORT RESPONSE You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.
   a. Draw and label a diagram to represent the situation.
   b. How far off the ground is the kite if you hold the spool 5 feet off the ground? Describe how the height where you hold the spool affects the height of the kite.

36. MULTI-STEP PROBLEM You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.
   a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window?
   b. The bush is 6 feet tall. Will your banner fit above the bush?
   c. What If? Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use?

37. ★ SHORT RESPONSE Nick uses the equation \( \sin 49° = \frac{x}{16} \) to find \( BC \) in \( \triangle ABC \). Tim uses the equation \( \cos 41° = \frac{x}{16} \). Which equation produces the correct answer? Explain.

38. TECHNOLOGY Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle?
39. **MULTIPLE REPRESENTATIONS** You are standing on a cliff 30 feet above an ocean. You see a sailboat on the ocean.
   a. **Drawing a Diagram** Draw and label a diagram of the situation.
   b. **Making a Table** Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.
   c. **Drawing a Graph** Graph the values you found in part (b), with the angle measures on the x-axis.
   d. **Making a Prediction** Predict the length of the line of sight when the angle of depression is 30°.

40. **ALGEBRA** If ΔEQU is equilateral and ΔRGT is a right triangle with RG = 2, RT = 1, and m∠T = 90°, show that sin E = cos G.

41. **CHALLENGE** Make a conjecture about the relationship between sine and cosine values.
   a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle.
   b. Compare the sine and cosine values. What pattern(s) do you notice?
   c. Make a conjecture about the sine and cosine values in part (b).
   d. Is the conjecture in part (c) true for right triangles that are not special right triangles? Explain.

### Mixed Review

**Rewrite the equation so that x is a function of y. (p. 877)**

42. \( y = \sqrt{x} \)  
43. \( y = 3x - 10 \)  
44. \( y = \frac{x}{9} \)

**Copy and complete the table. (p. 884)**

45.  
<table>
<thead>
<tr>
<th>x</th>
<th>( \sqrt{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td>?</td>
<td>1</td>
</tr>
<tr>
<td>?</td>
<td>( \sqrt{2} )</td>
</tr>
<tr>
<td>?</td>
<td>2</td>
</tr>
<tr>
<td>?</td>
<td>4</td>
</tr>
</tbody>
</table>

46.  
<table>
<thead>
<tr>
<th>x</th>
<th>( \frac{1}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>1</td>
</tr>
<tr>
<td>?</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>?</td>
<td>2</td>
</tr>
<tr>
<td>?</td>
<td>3</td>
</tr>
<tr>
<td>?</td>
<td>( \frac{2}{7} )</td>
</tr>
<tr>
<td>?</td>
<td>7</td>
</tr>
</tbody>
</table>

47.  
<table>
<thead>
<tr>
<th>x</th>
<th>( \frac{2}{7}x + 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td>?</td>
<td>2</td>
</tr>
<tr>
<td>?</td>
<td>6</td>
</tr>
<tr>
<td>?</td>
<td>8</td>
</tr>
<tr>
<td>?</td>
<td>10</td>
</tr>
</tbody>
</table>

48. Find the values of x and y in the triangle at the right. (p. 449)
Another Way to Solve Example 5, page 476

MUTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.

METHOD 1 Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 Find the measure of the third angle.

\[
26° + 90° + m\angle 3 = 180° \quad \text{Triangle Sum Theorem}
\]
\[
116° + m\angle 3 = 180° \quad \text{Combine like terms.}
\]
\[
m\angle 3 = 64° \quad \text{Subtract 116° from each side.}
\]

STEP 2 Use the cosine ratio to find the height of the ramp.

\[
\cos 64° = \frac{\text{adj.}}{\text{hyp.}} \quad \text{Write ratio for cosine of 64°.}
\]
\[
\cos 64° = \frac{x}{14} \quad \text{Substitute.}
\]
\[
14 \cdot \cos 64° = x \quad \text{Multiply each side by 14.}
\]
\[
6.1 \approx x \quad \text{Use a calculator to simplify.}
\]

The height is about 6.1 feet.

STEP 3 Use the Pythagorean Theorem to find the length of the base of the ramp.

\[
\text{(hypotenuse)}^2 = (\text{leg})^2 + (\text{leg})^2 \quad \text{Pythagorean Theorem}
\]
\[
14^2 = 6.1^2 + y^2 \quad \text{Substitute.}
\]
\[
196 = 37.21 + y^2 \quad \text{Multiply.}
\]
\[
158.79 = y^2 \quad \text{Subtract 37.21 from each side.}
\]
\[
12.6 \approx y \quad \text{Find the positive square root.}
\]

The length of the base is about 12.6 feet.
**Method 2**

Using a Tangent Ratio

Use the tangent ratio and \( h = 6.1 \) feet to find the length of the base of the ramp.

\[
\tan 26^\circ = \frac{\text{opp.}}{\text{adj.}}
\]

Write ratio for tangent of \( 26^\circ \).

\[
\tan 26^\circ = \frac{6.1}{y}
\]

Substitute.

\[y \cdot \tan 26^\circ = 61\]

Multiply each side by \( y \).

\[y = \frac{6.1}{\tan 26^\circ}\]

Divide each side by \( \tan 26^\circ \).

\[y \approx 12.5\]

Use a calculator to simplify.

The length of the base is about 12.5 feet.

Notice that when using the Pythagorean Theorem, the length of the base is 12.6 feet, but when using the tangent ratio, the length of the base is 12.5 feet. The tenth of a foot difference is due to the rounding error introduced when finding the height of the ramp and using that rounded value to calculate the length of the base.

**Practice**

1. **WHAT IF?** Suppose the length of the skateboard ramp is 20 feet. Find the height and base of the ramp.

2. **SWIMMER** The angle of elevation from the swimmer to the lifeguard is \( 35^\circ \). Find the distance \( x \) from the swimmer to the base of the lifeguard chair. Find the distance \( y \) from the swimmer to the lifeguard.

3. **ALGEBRA** Use the triangle below to write three different equations you can use to find the unknown leg length.

4. **SHORT RESPONSE** Describe how you would decide whether to use the Pythagorean Theorem or trigonometric ratios to find the lengths of unknown sides of a right triangle.

5. **ERROR ANALYSIS** Explain why the student’s statement is incorrect. Write a correct statement for the cosine of the angle.

6. **EXTENDED RESPONSE** You want to find the height of a tree in your yard. The tree’s shadow is 15 feet long and you measure the angle of elevation from the end of the shadow to the top of tree to be \( 75^\circ \).

   a. Find the height of the tree. Explain the method you chose to solve the problem.

   b. What else would you need to know to solve this problem using similar triangles.

   c. Explain why you cannot use the sine ratio to find the height of the tree.
To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:
- Two side lengths
- One side length and the measure of one acute angle

In Lessons 7.5 and 7.6, you learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

### KEY CONCEPT

#### Inverse Trigonometric Ratios

Let \( \angle A \) be an acute angle.

**Inverse Tangent** If \( \tan A = x \), then \( \tan^{-1} x = m \angle A \).

**Inverse Sine** If \( \sin A = y \), then \( \sin^{-1} y = m \angle A \).

**Inverse Cosine** If \( \cos A = z \), then \( \cos^{-1} z = m \angle A \).

### EXAMPLE 1

**Use an inverse tangent to find an angle measure**

Use a calculator to approximate the measure of \( \angle A \) to the nearest tenth of a degree.

![Diagram of a right triangle with sides 15 and 20]

**Solution**

Because \( \tan A = \frac{15}{20} = \frac{3}{4} = 0.75 \), \( \tan^{-1} 0.75 = m \angle A \). Use a calculator.

\[ \tan^{-1} 0.75 \approx 36.86989765 \cdots \]

So, the measure of \( \angle A \) is approximately 36.9°.
EXAMPLE 2  Use an inverse sine and an inverse cosine

Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.87$ 

Solution

a. $m\angle A = \sin^{-1} 0.87 \approx 60.5^\circ$

b. $\cos B = 0.15$

Solution

b. $m\angle B = \cos^{-1} 0.15 \approx 81.4^\circ$

EXAMPLE 3  Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.

Solution

STEP 1  Find $m\angle B$ by using the Triangle Sum Theorem.

$180^\circ = 90^\circ + 42^\circ + m\angle B$

$48^\circ = m\angle B$

STEP 2  Approximate $BC$ by using a tangent ratio.

$\tan 42^\circ = \frac{BC}{70}$ 

Write ratio for tangent of $42^\circ$.

70 $\cdot$ tan $42^\circ = BC$ 

Multiply each side by 70.

70 $\cdot$ 0.9004 $\approx BC$ 

Approximate tan $42^\circ$.

63 $\approx BC$ 

Simplify and round answer.

STEP 3  Approximate $AB$ using a cosine ratio.

$\cos 42^\circ = \frac{70}{AB}$ 

Write ratio for cosine of $42^\circ$.

$AB \cdot \cos 42^\circ = 70$ 

Multiply each side by $AB$.

$AB = \frac{70}{\cos 42^\circ}$ 

Divide each side by $\cos 42^\circ$.

$AB \approx \frac{70}{0.7431}$ 

Use a calculator to find $\cos 42^\circ$.

$AB \approx 94.2$ 

Simplify.

The angle measures are $42^\circ$, $48^\circ$, and $90^\circ$. The side lengths are 70 feet, about 63 feet, and about 94 feet.
**Example 4** Solve a real-world problem

**Theater Design** Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?

**Solution**

Use the sine and inverse sine ratios to find the degree measure $x$ of the rake.

$$\sin x^\circ = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x = \sin^{-1} 0.0667 \approx 3.824$$

The rake is about 3.8°, so it is within the suggested range of 5° or less.

---

**Guided Practice** for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse.

4. **What if?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? Explain.

---

**7.7 Exercises**

**Skill Practice**

1. **Vocabulary** Copy and complete: To solve a right triangle means to find the measures of all of its __?__ and __?__.

2. **Writing** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem.

3. **Using Inverse Tangents** Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

4. **Worked-Out Solutions** on p. W51 for Exs. 5, 13, and 35

5. **Standardized Test Practice** Exs. 2, 9, 29, 30, 35, 40, and 41

6. **Multiple Representations** Ex. 39
USING INVERSE SINES AND COSINES Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

6. $5 \quad 11 \quad C \quad A \quad B$

7. $6 \quad 10 \quad A \quad C \quad B$

8. $12 \quad A \quad 7 \quad B$

9. ★ MULTIPLE CHOICE Which expression is correct?
   - $\text{A} \quad \sin^{-1} \frac{IL}{JK} = m \angle J$
   - $\text{B} \quad \tan^{-1} \frac{KL}{IL} = m \angle J$
   - $\text{C} \quad \cos^{-1} \frac{IL}{JK} = m \angle K$
   - $\text{D} \quad \sin^{-1} \frac{IL}{KL} = m \angle K$

SOLVING RIGHT TRIANGLES Solve the right triangle. Round decimal answers to the nearest tenth.

10. $8 \quad K \quad M \quad 40^\circ \quad L$

11. $Q \quad 65^\circ \quad 10 \quad N \quad P$

12. $R \quad 57^\circ \quad 15 \quad S \quad T$

13. $B \quad 12 \quad A \quad 9 \quad C$

14. $E \quad 3 \quad 9 \quad D \quad F$

15. $G \quad 14 \quad 14 \quad H$

16. $C \quad 5.2 \quad 43.6^\circ \quad A \quad B$

17. $8 \quad E \quad 14 \quad 3 \quad D \quad 8$

18. $29.9^\circ \quad G \quad 10 \quad E \quad J \quad H$

ERROR ANALYSIS Describe and correct the student's error in using an inverse trigonometric ratio.

19. $\sin^{-1} \frac{7}{WY} = 36^\circ$

20. $\cos^{-1} \frac{8}{15} = m \angle T$

CALCULATOR Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.

21. $\sin A = 0.5$

22. $\sin A = 0.75$

23. $\cos A = 0.33$

24. $\cos A = 0.64$

25. $\tan A = 1.0$

26. $\tan A = 0.28$

27. $\sin A = 0.19$

28. $\cos A = 0.81$
29. ★ MULTIPLE CHOICE Which additional information would not be enough to solve \( \triangle PRQ \)?

A. \( m \angle P \) and \( PR \)  
B. \( m \angle P \) and \( m \angle R \)  
C. \( PQ \) and \( PR \)  
D. \( m \angle P \) and \( PQ \)

30. ★ WRITING Explain why it is incorrect to say that \( \tan^{-1} x = \frac{1}{\tan x} \).

31. SPECIAL RIGHT TRIANGLES If \( \sin A = \frac{1}{2} \sqrt{2} \), what is \( m \angle A \)? If \( \sin B = \frac{1}{2} \sqrt{3} \), what is \( m \angle B \)?

32. TRIGONOMETRIC VALUES Use the Table of Trigonometric Ratios on page 925 to answer the questions.

a. What angles have nearly the same sine and tangent values?

b. What angle has the greatest difference in its sine and tangent value?

c. What angle has a tangent value that is double its sine value?

d. Is \( \sin 2x \) equal to \( 2 \cdot \sin x \)?

33. CHALLENGE The perimeter of rectangle \( ABCD \) is 16 centimeters, and the ratio of its width to its length is 1 : 3. Segment \( BD \) divides the rectangle into two congruent triangles. Find the side lengths and angle measures of one of these triangles.

34. SOCCER A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick?

35. ★ SHORT RESPONSE You are standing on a footbridge in a city park that is 12 feet high above a pond. You look down and see a duck in the water 7 feet away from the footbridge. What is the angle of depression? Explain your reasoning.

36. CLAY In order to unload clay easily, the body of a dump truck must be elevated to at least \( 55^\circ \). If the body of the dump truck is 14 feet long and has been raised 10 feet, will the clay pour out easily?

37. REASONING For \( \triangle ABC \) shown, each of the expressions \( \sin^{-1} \frac{BC}{AB} \), \( \cos^{-1} \frac{AC}{AB} \), and \( \tan^{-1} \frac{BC}{AC} \) can be used to approximate the measure of \( \angle A \). Which expression would you choose? Explain your choice.
38. **MULTI-STEP PROBLEM** You are standing on a plateau that is 800 feet above a basin where you can see two hikers.

   ![Diagram of plateau and hikers](image)

   a. If the angle of depression from your line of sight to the hiker at $B$ is $25^\circ$, how far is the hiker from the base of the plateau?

   b. If the angle of depression from your line of sight to the hiker at $C$ is $15^\circ$, how far is the hiker from the base of the plateau?

   c. How far apart are the two hikers? *Explain.*

39. **MULTIPLE REPRESENTATIONS** A local ranch offers trail rides to the public. It has a variety of different sized saddles to meet the needs of horse and rider. You are going to build saddle racks that are 11 inches high. To save wood, you decide to make each rack fit each saddle.

   a. **Making a Table** The lengths of the saddles range from 20 inches to 27 inches. Make a table showing the saddle rack length $x$ and the measure of the adjacent angle $y^\circ$.

   b. **Drawing a Graph** Use your table to draw a scatterplot.

   c. **Making a Conjecture** Make a conjecture about the relationship between the length of the rack and the angle needed.

40. **★ OPEN-ENDED MATH** Describe a real-world problem you could solve using a trigonometric ratio.

41. **★ EXTENDED RESPONSE** Your town is building a wind generator to create electricity for your school. The builder wants your geometry class to make sure that the guy wires are placed so that the tower is secure. By safety guidelines, the distance along the ground from the tower to the guy wire’s connection with the ground should be between 50% to 75% of the height of the guy wire’s connection with the tower.

   a. The tower is 64 feet tall. The builders plan to have the distance along the ground from the tower to the guy wire’s connection with the ground be 60% of the height of the tower. How far apart are the tower and the ground connection of the wire?

   b. How long will a guy wire need to be that is attached 60 feet above the ground?

   c. How long will a guy wire need to be that is attached 30 feet above the ground?

   d. Find the angle of elevation of each wire. Are the right triangles formed by the ground, tower, and wires *congruent, similar,* or *neither? Explain.*

   e. *Explain* which trigonometric ratios you used to solve the problem.
42. **CHALLENGE** Use the diagram of \( \triangle ABC \).

**GIVEN** \( \triangle ABC \) with altitude \( CD \).

**PROVE** \( \sin A \over a = \sin B \over b \)

---

**MIXED REVIEW**

43. Copy and complete the table. (p. 42)

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Type of polygon</th>
<th>Number of sides</th>
<th>Type of polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>?</td>
<td>10</td>
<td>?</td>
</tr>
<tr>
<td>12</td>
<td>?</td>
<td>9</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>Octagon</td>
<td>10</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>Triangle</td>
<td>9</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>?</td>
<td>?</td>
<td>Hexagon</td>
</tr>
</tbody>
</table>

A point on an image and the transformation are given. Find the corresponding point on the original figure. (p. 272)

44. Point on image: \((5, 1);\) translation: \((x, y) \rightarrow (x + 3, y - 2)\)

45. Point on image: \((4, -6);\) reflection: \((x, y) \rightarrow (x, -y)\)

46. Point on image: \((-2, 3);\) translation: \((x, y) \rightarrow (x - 5, y + 7)\)

Draw a dilation of the polygon with the given vertices using the given scale factor \( k \). (p. 409)

47. \( A(2, 2), B(-1, -3), C(5, -3); k = 2 \)

48. \( A(-4, -2), B(-2, 4), C(3, 6), D(6, 3); k = \frac{1}{2} \)

---

**QUIZ for Lessons 7.5–7.7**

Find the value of \( x \) to the nearest tenth.

1. (p. 466)

2. (p. 473)

3. (p. 473)

Solve the right triangle. Round decimal answers to the nearest tenth. (p. 483)

4.

5.

6.
Law of Sines and Law of Cosines

**GOAL** Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in any triangle.

**KEY CONCEPT**

**Law of Sines**

If \( \triangle ABC \) has sides of length \( a \), \( b \), and \( c \) as shown, then

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.
\]

**EXAMPLE 1** Find a distance using Law of Sines

**DISTANCE** Use the information in the diagram to determine how much closer you live to the music store than your friend does.

**Solution**

**STEP 1** Use the Law of Sines to find the distance \( a \) from your friend’s home to the music store.

\[
\frac{\sin A}{a} = \frac{\sin C}{c}
\]

Write Law of Sines.

\[
\frac{\sin 81^\circ}{a} = \frac{\sin 34^\circ}{1.5}
\]

Substitute.

\[
a \approx 2.6
\]

Solve for \( a \).

**STEP 2** Use the Law of Sines to find the distance \( b \) from your home to the music store.

\[
\frac{\sin B}{b} = \frac{\sin C}{c}
\]

Write Law of Sines.

\[
\frac{\sin 65^\circ}{b} = \frac{\sin 34^\circ}{1.5}
\]

Substitute.

\[
b \approx 2.4
\]

Solve for \( b \).

**STEP 3** Subtract the distances.

\[
a - b \approx 2.6 - 2.4 = 0.2
\]

You live about 0.2 miles closer to the music store.
LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

**KEY CONCEPT**

**Law of Cosines**

If \( \triangle ABC \) has sides of length \( a \), \( b \), and \( c \), then:

\[
\begin{align*}
    a^2 &= b^2 + c^2 - 2bc \cos A \\
    b^2 &= a^2 + c^2 - 2ac \cos B \\
    c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

**EXAMPLE 2** Find an angle measure using Law of Cosines

In \( \triangle ABC \) at the right, \( a = 11 \text{ cm} \), \( b = 17 \text{ cm} \), and \( c = 19 \text{ cm} \). Find \( m \angle C \).

**Solution**

\[
\begin{align*}
    c^2 &= a^2 + b^2 - 2ab \cos C \\
    19^2 &= 11^2 + 17^2 - 2(11)(17) \cos C \\
    0.1310 &= \cos C \\
    m \angle C &= 82^\circ
\end{align*}
\]

**PRACTICE**

1. **LAW OF SINES** Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.
   - 1.
   - 2.
   - 3.

2. **LAW OF COSINES** Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.
   - 4.
   - 5.
   - 6.

7. **DISTANCE** Use the diagram at the right. Find the straight distance between the zoo and movie theater.
1. **MULTI-STEP PROBLEM** A reach stacker is a vehicle used to lift objects and move them between ships and land.

   a. The vehicle’s arm is 10.9 meters long. The maximum measure of $\angle A$ is 60°. What is the greatest height $h$ the arm can reach if the vehicle is 3.6 meters tall?

   b. The vehicle’s arm can extend to be 16.4 meters long. What is the greatest height its extended arm can reach?

   c. What is the difference between the two heights the arm can reach above the ground?

2. **EXTENDED RESPONSE** You and a friend are standing the same distance from the edge of a canyon. Your friend looks directly across the canyon at a rock. You stand 10 meters from your friend and estimate the angle between your friend and the rock to be 85°.

   a. Sketch the situation.

   b. Explain how to find the distance across the canyon.

   c. Suppose the actual angle measure is 87°. How far off is your estimate of the distance?

3. **SHORT RESPONSE** The international rules of basketball state the rim of the net should be 3.05 meters above the ground. If your line of sight to the rim is 34° and you are 1.7 meters tall, what is the distance from you to the rim? Explain your reasoning.

4. **GRIDDED ANSWER** The specifications for a yield ahead pavement marking are shown. Find the height $h$ in feet of this isosceles triangle.

5. **EXTENDED RESPONSE** Use the diagram to answer the questions.

   a. Solve for $x$. Explain the method you chose.

   b. Find $m\angle ABC$. Explain the method you chose.

   c. Explain a different method for finding each of your answers in parts (a) and (b).

6. **SHORT RESPONSE** The triangle on the staircase below has a 52° angle and the distance along the stairs is 14 feet. What is the height $h$ of the staircase? What is the length $b$ of the base of the staircase?

7. **GRIDDED ANSWER** The base of an isosceles triangle is 70 centimeters long. The altitude to the base is 75 centimeters long. Find the measure of a base angle to the nearest degree.
BIG IDEAS

Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse \( c \) is equal to the sum of the squares of the lengths of the legs \( a \) and \( b \), so that \( c^2 = a^2 + b^2 \).

The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.

If \( c^2 = a^2 + b^2 \), then \( \angle C = 90^\circ \) and \( \triangle ABC \) is a right triangle.

If \( c^2 < a^2 + b^2 \), then \( \angle C < 90^\circ \) and \( \triangle ABC \) is an acute triangle.

If \( c^2 > a^2 + b^2 \), then \( \angle C > 90^\circ \) and \( \triangle ABC \) is an obtuse triangle.

Using Special Relationships in Right Triangles

GEOMETRIC MEAN In right \( \triangle ABC \), altitude \( CD \) forms two smaller triangles so that \( \triangle CBD \sim \triangle ACD \sim \triangle ABC \).

Also, \( \frac{BD}{CD} = \frac{CD}{AD} \), \( \frac{AB}{BC} = \frac{CB}{DB} \), and \( \frac{AB}{AC} = \frac{AC}{AD} \).

SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle

- hypotenuse = leg \( \cdot \sqrt{2} \)

30°-60°-90° Triangle

- hypotenuse = 2 \( \cdot \) shorter leg
- longer leg = shorter leg \( \cdot \sqrt{3} \)

Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of \( \tan \angle x \), \( \sin \angle x \), and \( \cos \angle x \) depend only on the angle measure and not on the side length.

\[
\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC}
\]

\[
\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB}
\]

\[
\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB}
\]

\[
\tan^{-1} \frac{BC}{AC} = m\angle A
\]

\[
\sin^{-1} \frac{BC}{AB} = m\angle A
\]

\[
\cos^{-1} \frac{AC}{AB} = m\angle A
\]
CHAPTER REVIEW

REVIEW KEY VOCABULARY

- Pythagorean triple, p. 435
- Trigonometric ratio, p. 466
- Tangent, p. 466
- Sine, p. 473
- Cosine, p. 473
- Angle of elevation, p. 475
- Angle of depression, p. 475
- Solve a right triangle, p. 483
- Inverse tangent, p. 483
- Inverse sine, p. 483
- Inverse cosine, p. 483

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers $a$, $b$, and $c$ that satisfy the equation $?$.

2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?

3. **WRITING** Describe the difference between an angle of depression and an angle of elevation.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem

**Example**

Find the value of $x$.

Because $x$ is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$$

$x^2 = 15^2 + 20^2$

$x^2 = 625$

$x = 25$

**Exercises**

Find the unknown side length $x$.

4. 

5. 

6. 

For a list of postulates and theorems, see pp. 926–931.

For a list of postulates and theorems, see pp. 926–931.

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- Multi-Language Glossary
- Vocabulary practice
Use the Converse of the Pythagorean Theorem

**Example**

Tell whether the given triangle is a right triangle.

Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

12² ≠ (√65)² + 9²

144 ≠ 65 + 81

144 < 146

The triangle is not a right triangle. It is an acute triangle.

**Exercises**

Classify the triangle formed by the side lengths as **acute**, **right**, or **obtuse**.

7. 6, 8, 9
8. 4, 2, 5
9. 10, $2\sqrt{3}$, 6$\sqrt{3}$
10. 15, 20, 15
11. 3, 3, $3\sqrt{2}$
12. 13, 18, $3\sqrt{55}$

Use Similar Right Triangles

**Example**

Find the value of $x$.

By Theorem 7.6, you know that 4 is the geometric mean of $x$ and 2.

\[
\frac{x}{4} = \frac{2}{2} \quad \text{Write a proportion.}
\]

\[2x = 16 \quad \text{Cross Products Property}
\]

\[x = 8 \quad \text{Divide.}
\]

**Exercises**

Find the value of $x$.

16. 17. 18.
7.4 Special Right Triangles

**Example**

Find the length of the hypotenuse.

By the Triangle Sum Theorem, the measure of the third angle must be $45^\circ$. Then the triangle is a $45^\circ$-$45^\circ$-$90^\circ$ triangle.

\[
\text{hypotenuse} = \text{leg} \times \sqrt{2} \\
x = 10\sqrt{2}
\]

**Exercises**

Find the value of $x$. Write your answer in simplest radical form.

19. 

20. 

21.

7.5 Apply the Tangent Ratio

**Example**

Find the value of $x$.

\[
\tan 37^\circ = \frac{\text{opp.}}{\text{adj.}} \\
\tan 37^\circ = \frac{x}{8} \\
8 \cdot \tan 37^\circ = x \\
6 = x
\]

**Exercises**

In Exercises 22 and 23, use the diagram.

22. The angle between the bottom of a fence and the top of a tree is $75^\circ$. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

23. In Exercise 22, how tall is the tree if the angle is $55^\circ$?

Find the value of $x$ to the nearest tenth.

24. 

25. 

26.
Apply the Sine and Cosine Ratios

**Example**

Find \( \sin A \) and \( \sin B \).

\[
\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824
\]

\[
\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706
\]

**Exercises**

Find \( \sin X \) and \( \cos X \). Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

27. \[
\begin{align*}
\text{X} & \quad \text{Y} \\
3 & \quad 5 \\
\text{Y} & \quad \text{X}
\end{align*}
\]

28. \[
\begin{align*}
\text{Y} & \quad \text{Z} \\
149 & \quad \text{X}
\end{align*}
\]

29. \[
\begin{align*}
\text{Y} & \quad \text{X} \\
48 & \quad 55 \\
\text{X} & \quad \text{Z}
\end{align*}
\]

Solve Right Triangles

**Example**

Use a calculator to approximate the measure of \( \angle A \) to the nearest tenth of a degree.

Because \( \tan A = \frac{18}{12} = \frac{3}{2} = 1.5 \), \( \tan^{-1} 1.5 = m \angle A \).

Use a calculator to evaluate this expression.

\[
\tan^{-1} 1.5 \approx 56.3099324 \ldots
\]

So, the measure of \( \angle A \) is approximately \( 56.3^{\circ} \).

**Exercises**

Solve the right triangle. Round decimal answers to the nearest tenth.

30. \[
\begin{align*}
\text{B} & \quad 15 \\
\text{C} & \quad 10 \\
\text{A}
\end{align*}
\]

31. \[
\begin{align*}
\text{N} & \quad 37^{\circ} \\
\text{M} & \quad 6 \\
\text{L}
\end{align*}
\]

32. \[
\begin{align*}
\text{X} & \quad 25 \\
\text{Y} & \quad 18 \\
\text{Z}
\end{align*}
\]

33. Find the measures of \( \angle GED \), \( \angle GEF \), and \( \angle EFG \). Find the lengths of \( EG \), \( DF \), \( EF \).
Find the value of $x$. Write your answer in simplest radical form.

1. \[ \begin{array}{c}
    \text{20} \\
    \text{12} \\
    x
\end{array} \]

2. \[ \begin{array}{c}
    x \\
    9 \\
    13
\end{array} \]

3. \[ \begin{array}{c}
    21 \\
    x \\
    15
\end{array} \]

Classify the triangle as **acute**, **right**, or **obtuse**.

4. \(5, 15, 5\sqrt{10}\)
5. \(4.3, 6.7, 8.2\)
6. \(5, 7, 8\)

Find the value of $x$. Round decimal answers to the nearest tenth.

7. \[ \begin{array}{c}
    x \\
    20 \\
    5
\end{array} \]

8. \[ \begin{array}{c}
    24 \\
    x \\
    10
\end{array} \]

9. \[ \begin{array}{c}
    12 \\
    3 \\
    x
\end{array} \]

Find the value of each variable. Write your answer in simplest radical form.

10. \[ \begin{array}{c}
    y \\
    x
\end{array} \]

11. \[ \begin{array}{c}
    x \\
    24
\end{array} \]

12. \[ \begin{array}{c}
    y \\
    x
\end{array} \]

Solve the right triangle. Round decimal answers to the nearest tenth.

13. \[ \begin{array}{c}
    A \\
    11 \\
    B \\
    5 \\
    C
\end{array} \]

14. \[ \begin{array}{c}
    E \\
    5.4 \\
    F \\
    9.2 \\
    D
\end{array} \]

15. \[ \begin{array}{c}
    G \\
    H \\
    14 \\
    J
\end{array} \]

16. **FLAGPOLE** Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole.

17. **HILLS** The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill?
The graph of \( y = ax^2 + bx + c \) is a parabola that opens upward if \( a > 0 \) and opens downward if \( a < 0 \). The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} \). The axis of symmetry is the vertical line \( x = -\frac{b}{2a} \).

**Graph and Solve Quadratic Equations**

The graph of \( y = ax^2 + bx + c \) is a parabola that opens upward if \( a > 0 \) and opens downward if \( a < 0 \). The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} \). The axis of symmetry is the vertical line \( x = -\frac{b}{2a} \).

**Exercises**

Graph the quadratic function. Label the vertex and axis of symmetry.

1. \( y = x^2 - 6x + 8 \)
2. \( y = -x^2 - 4x + 2 \)
3. \( y = 2x^2 - x - 1 \)
4. \( y = 3x^2 - 9x + 2 \)
5. \( y = \frac{1}{2}x^2 - x + 3 \)
6. \( y = -4x^2 + 6x - 5 \)

Solve the quadratic equation by graphing. Check solutions algebraically.

7. \( x^2 = x + 6 \)
8. \( 4x + 4 = -x^2 \)
9. \( 2x^2 = -8 \)
10. \( 3x^2 + 2 = 14 \)
11. \( -x^2 + 4x - 5 = 0 \)
12. \( 2x - x^2 = -15 \)
13. \( \frac{1}{4}x^2 = 2x \)
14. \( x^2 + 3x = 4 \)
15. \( x^2 + 8 = 6x \)
16. \( x^2 = 9x - 1 \)
17. \( -25 = x^2 + 10x \)
18. \( x^2 + 6x = 0 \)
MULTIPLE CHOICE QUESTIONS

You ride your bike at an average speed of 10 miles per hour. How long does it take you to ride one time around the triangular park shown in the diagram?

A 0.1 h  B 0.2 h  C 0.3 h  D 0.4 h

**METHOD 1**

**SOLVE DIRECTLY** The park is a right triangle. Use the Pythagorean Theorem to find KL. Find the perimeter of \(\triangle JKL\). Then find how long it takes to ride around the park.

**STEP 1** Find KL. Use the Pythagorean Theorem.

\[
JK^2 + KL^2 = JL^2
\]
\[
1.5^2 + KL^2 = 1.7^2
\]
\[
2.25 + KL^2 = 2.89
\]
\[
KL^2 = 0.64
\]
\[
KL = 0.8
\]

**STEP 2** Find the perimeter of \(\triangle JKL\).

\[
P = JK + JL + KL
\]
\[
= 1.5 + 1.7 + 0.8
\]
\[
= 4 \text{ mi}
\]

**STEP 3** Find the time \(t\) (in hours) it takes you to go around the park.

Rate \times Time = Distance

\[
(10 \text{ mi/h}) \times t = 4 \text{ mi}
\]
\[
t = 0.4 \text{ h}
\]

The correct answer is D. A B C D

**METHOD 2**

**ELIMINATE CHOICES** Another method is to find how far you can travel in the given times to eliminate choices that are not reasonable.

**STEP 1** Find how far you will travel in each of the given times. Use the formula \(rt = d\).

Choice A: \(0.1(10) = 1 \text{ mi}\)
Choice B: \(0.2(10) = 2 \text{ mi}\)
Choice C: \(0.3(10) = 3 \text{ mi}\)
Choice D: \(0.4(10) = 4 \text{ mi}\)

The distance around two sides of the park is \(1.5 + 1.7 = 3.2 \text{ mi}\). But you need to travel around all three sides, which is longer.

Since \(1 < 3.2, 2 < 3.2, \) and \(3 < 3.2\). You can eliminate choices A, B, and C.

**STEP 2** Check that D is the correct answer. If the distance around the park is 4 miles, then

\[
KL = 4 - JK - JL
\]
\[
= 4 - 1.5 - 1.7 = 0.8 \text{ mi}
\]

Apply the Converse of the Pythagorean Theorem.

\[
0.8^2 + 1.5^2 \neq 1.7^2
\]
\[
0.64 + 2.25 \neq 2.89
\]
\[
2.89 = 2.89 \checkmark
\]

The correct answer is D. A B C D
What is the height of $\triangle WXY$?

- **A** 4
- **B** $4\sqrt{3}$
- **C** 8
- **D** $8\sqrt{3}$

**METHOD 1**

**SOLVE DIRECTLY** Draw altitude $XZ$ to form two congruent $30^\circ$-$60^\circ$-$90^\circ$ triangles.

Let $h$ be the length of the longer leg of $\triangle XZY$. The length of the shorter leg is 4.

\[
\text{longer leg} = \sqrt{3} \cdot \text{shorter leg} \\
\text{longer leg} = 4\sqrt{3} \\
h = 4\sqrt{3}
\]

The correct answer is B. (A) (B) (C) (D)

**METHOD 2**

**ELIMINATE CHOICES** Another method is to use theorems about triangles to eliminate incorrect choices. Draw altitude $XZ$ to form two congruent right triangles.

Consider $\triangle XZW$. By the Triangle Inequality, $XW < WZ + XZ$. So, $8 < 4 + XZ$ and $XZ > 4$. You can eliminate choice A. Also, $XZ$ must be less than the hypotenuse of $\triangle XWZ$. You can eliminate choices C and D.

The correct answer is B. (A) (B) (C) (D)

---

**PRACTICE**

**Explain why you can eliminate the highlighted answer choice.**

1. In the figure shown, what is the length of $EF$?

- **A** 9
- **B** $9\sqrt{2}$
- **C** 18
- **D** $9\sqrt{5}$

2. Which of the following lengths are side lengths of a right triangle?

- **A** $2, 21, 23$
- **B** 3, 4, 5
- **C** 9, 16, 18
- **D** 11, 16, 61

3. In $\triangle PQR$, $PQ = QR = 13$ and $PR = 10$. What is the length of the altitude drawn from vertex $Q$?

- **A** 10
- **B** 11
- **C** 12
- **D** $12$
1. Which expression gives the correct length for $XW$ in the diagram below?

   \[ A \quad 5 + 5\sqrt{2} \quad B \quad 5 + 5\sqrt{3} \quad C \quad 5\sqrt{3} + 5\sqrt{2} \quad D \quad 5 + 10 \]

2. The area of $\triangle EFG$ is 400 square meters. To the nearest tenth of a meter, what is the length of side $EG$?

   \[ A \quad 10.0 \text{ meters} \quad B \quad 20.0 \text{ meters} \quad C \quad 44.7 \text{ meters} \quad D \quad 56.7 \text{ meters} \]

3. Which expression can be used to find the value of $x$ in the diagram below?

   \[ A \quad \tan 29^\circ = \frac{x}{17} \quad B \quad \cos 29^\circ = \frac{x}{17} \quad C \quad \tan 61^\circ = \frac{x}{17} \quad D \quad \cos 61^\circ = \frac{x}{17} \]

4. A fire station, a police station, and a hospital are not positioned in a straight line. The distance from the police station to the fire station is 4 miles. The distance from the fire station to the hospital is 3 miles. Which of the following could not be the distance from the police station to the hospital?

   \[ A \quad 1 \text{ mile} \quad B \quad 2 \text{ miles} \quad C \quad 5 \text{ miles} \quad D \quad 6 \text{ miles} \]

5. It takes 14 minutes to walk from your house to your friend’s house on the path shown in red. If you walk at the same speed, about how many minutes will it take on the path shown in blue?

   \[ A \quad 6 \text{ minutes} \quad B \quad 8 \text{ minutes} \quad C \quad 10 \text{ minutes} \quad D \quad 13 \text{ minutes} \]

6. Which equation can be used to find $QR$ in the diagram below?

   \[ A \quad \frac{QR}{15} = \frac{15}{7} \quad B \quad \frac{15}{QR} = \frac{QR}{8} \quad C \quad QR = \sqrt{15^2 + 27^2} \quad D \quad \frac{QR}{7} = \frac{7}{15} \]

7. Stitches are sewn along the black line segments in the potholder shown below. There are 10 stitches per inch. Which is the closest estimate of the number of stitches used?

   \[ A \quad 480 \quad B \quad 550 \quad C \quad 656 \quad D \quad 700 \]
8. A design on a T-shirt is made of a square and four equilateral triangles. The side length of the square is 4 inches. Find the distance (in inches) from point A to point B. Round to the nearest tenth.

9. Use the diagram below. Find KM to the nearest tenth of a unit.

10. The diagram shows the side of a set of stairs. In the diagram, the smaller right triangles are congruent. Explain how to find the lengths x, y, and z.

11. You drive due north from Dalton to Bristol. Next, you drive from Bristol to Hilldale. Finally, you drive from Hilldale to Dalton. Is Hilldale due west of Bristol? Explain.

12. The design for part of a water ride at an amusement park is shown. The ride carries people up a track along ramp AB. Then riders travel down a water chute along ramp BC.
   a. How high is the ride above point D? Explain.
   b. What is the total distance from point A to point B to point C? Explain.

13. A formula for the area A of a triangle is Heron’s Formula.
   For a triangle with side lengths EF, FG, and EG, the formula is
   \[ A = \sqrt{s(s - EF)(s - FG)(s - EG)}, \] where \( s = \frac{1}{2}(EF + FG + EG). \)
   a. In \( \triangle EFG \) shown, \( EF = FG = 15, \) and \( EG = 18. \) Use Heron’s formula to find the area of \( \triangle EFG. \) Round to the nearest tenth.
   b. Use the formula \( A = \frac{1}{2}bh \) to find the area of \( \triangle EFG. \) Round to the nearest tenth.
   c. Use Heron’s formula to justify that the area of an equilateral triangle with side length \( x \) is \( A = \frac{x^2\sqrt{3}}{4}. \)