4 Congruent Triangles

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 4: classifying angles, solving linear equations, finding midpoints, and using angle relationships.

Prerequisite Skills

VOCABULARY CHECK

Classify the angle as *acute*, *obtuse*, *right*, or *straight*.

1. $m\angle A = 115°$ **2.** $m\angle B = 90°$ **3.** $m\angle C = 35°$ **4.** $m\angle D = 95°$

SKILLS AND ALGEBRA CHECK

Solve the equation. *(Review p. 65 for 4.1, 4.2.)*

5. $70 + 2y = 180$ **6.** $2x = 5x - 54$ **7.** $40 + x + 65 = 180$

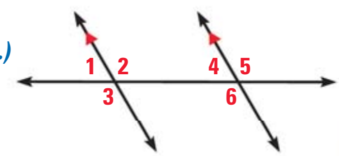
Find the coordinates of the midpoint of $\overline{PQ}$. *(Review p. 15 for 4.3.)*

8. $P(2, -5), Q(-1, -2)$ **9.** $P(-4, 7), Q(1, -5)$ **10.** $P(h, k), Q(h, 0)$

Name the theorem or postulate that justifies the statement about the diagram. *(Review p. 154 for 4.3–4.5.)*

11. $\angle 2 \cong \angle 3$ **12.** $\angle 1 \cong \angle 4$

13. $\angle 2 \cong \angle 6$ **14.** $\angle 3 \cong \angle 5$



@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 4, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 281. You will also use the key vocabulary listed below.

Big Ideas

1. Classifying triangles by sides and angles
2. Proving that triangles are congruent
3. Using coordinate geometry to investigate triangle relationships

KEY VOCABULARY

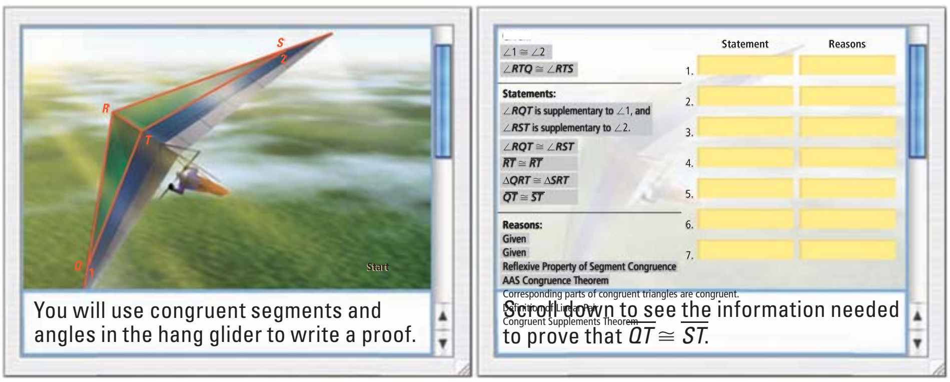
- triangle, *p. 217*
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles, *p. 218*
- exterior angles, *p. 218*
- corollary, *p. 220*
- congruent figures, *p. 225*
- corresponding parts, *p. 225*
- right triangle, *p. 241*
  legs, hypotenuse
- flow proof, *p. 250*
- isosceles triangle, *p. 264*
  legs, vertex angle, base, base angles
- transformation, *p. 272*
  translation, reflection, rotation

Why?

Triangles are used to add strength to structures in real-world situations. For example, the frame of a hang glider involves several triangles.

Animated Geometry

The animation illustrated below for Example 1 on page 256 helps you answer this question: What must be true about $\overline{QT}$ and $\overline{ST}$ for the hang glider to fly straight?



Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 4: pages 234, 242, 250, 257, and 274

Investigating Geometry ACTIVITY *Use before Lesson 4.1*

4.1 Angle Sums in Triangles

MATERIALS • paper • pencil • scissors • ruler

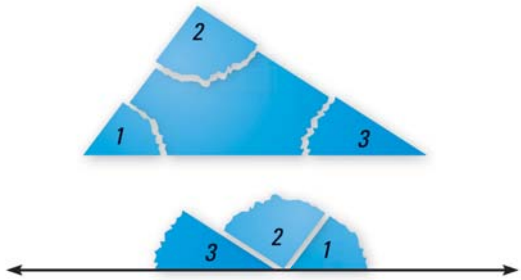
QUESTION **What are some relationships among the *interior angles* of a triangle and *exterior angles* of a triangle?**

EXPLORE 1 Find the sum of the measures of interior angles

STEP 1 ***Draw triangles*** Draw and cut out several different triangles.

STEP 2 ***Tear off corners*** For each triangle, tear off the three corners and place them next to each other, as shown in the diagram.

STEP 3 ***Make a conjecture*** Make a conjecture about the sum of the measures of the interior angles of a triangle.



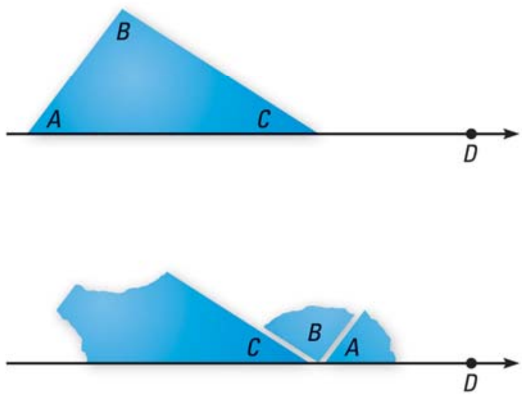
∠1, ∠2, and ∠3 are *interior angles*.

EXPLORE 2 Find the measure of an exterior angle of a triangle

STEP 1 ***Draw exterior angle*** Draw and cut out several different triangles. Place each triangle on a piece of paper and extend one side to form an *exterior angle*, as shown in the diagram.

STEP 2 ***Tear off corners*** For each triangle, tear off the corners that are not next to the exterior angle. Use them to fill the exterior angle, as shown.

STEP 3 ***Make a conjecture*** Make a conjecture about the relationship between the measure of an exterior angle of a triangle and the measures of the nonadjacent interior angles.



In the top figure, ∠*BCD* is an *exterior angle*.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Given the measures of two interior angles of a triangle, how can you find the measure of the third angle?
2. Draw several different triangles that each have one right angle. Show that the two acute angles of a right triangle are complementary.

4.1 Apply Triangle Sum Properties

Before You classified angles and found their measures.

Now You will classify triangles and find measures of their angles.

Why? So you can place actors on stage, as in Ex. 40.

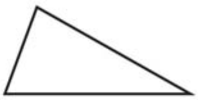
Key Vocabulary
- **triangle**
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- **interior angles**
- **exterior angles**
- **corollary to a theorem**

A **triangle** is a polygon with three sides. A triangle with vertices *A*, *B*, and *C* is called "triangle *ABC*" or "△*ABC*."

KEY CONCEPT *For Your Notebook*

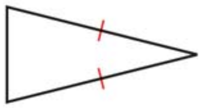
Classifying Triangles by Sides

Scalene Triangle



No congruent sides

Isosceles Triangle



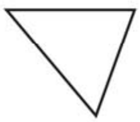
At least 2 congruent sides

Equilateral Triangle

3 congruent sides

Classifying Triangles by Angles

Acute Triangle

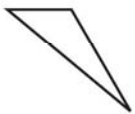


3 acute angles

Right Triangle

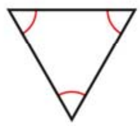
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



3 congruent angles

READ VOCABULARY
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

EXAMPLE 1 Classify triangles by sides and by angles

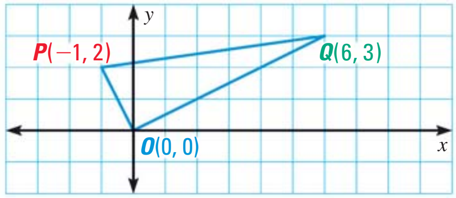
SUPPORT BEAMS Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.

Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°. It is an acute isosceles triangle.

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

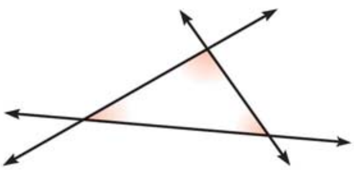
▶ Therefore, $\triangle PQO$ is a right scalene triangle.

✓ GUIDED PRACTICE for Examples 1 and 2

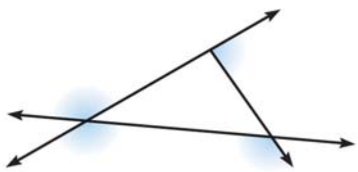
1. Draw an obtuse isosceles triangle and an acute scalene triangle.
2. Triangle ABC has the vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify it by its sides. Then determine if it is a right triangle.

ANGLES When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

READ DIAGRAMS
Each vertex has a *pair* of congruent exterior angles. However, it is common to show only *one* exterior angle at each vertex.



interior angles



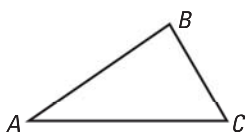
exterior angles

THEOREM *For Your Notebook*

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Proof: p. 219; Ex. 53, p. 224



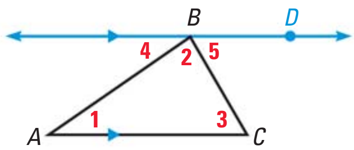
$m\angle A + m\angle B + m\angle C = 180°$

AUXILIARY LINES To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Plan in Action

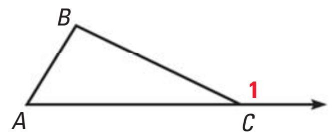
	STATEMENTS	REASONS
a.	1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	1. Parallel Postulate
b.	2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	2. Angle Addition Postulate and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. Alternate Interior Angles Theorem
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	4. Definition of congruent angles
c.	5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5. Substitution Property of Equality

THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

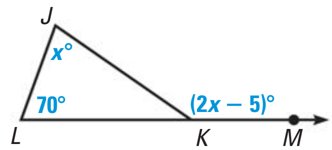
Proof: Ex. 50, p. 223



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Find an angle measure

ALGEBRA **Find $m\angle JKM$.**



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ **Apply the Exterior Angle Theorem.**

$x = 75$ **Solve for *x*.**

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

▶ The measure of $\angle JKM$ is 145°.

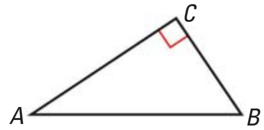
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY *For Your Notebook*

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Proof: Ex. 48, p. 223



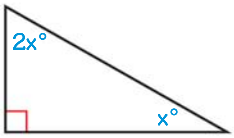
$m\angle A + m\angle B = 90°$

EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.

Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

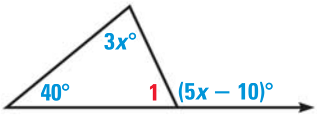
$x° + 2x° = 90°$ **Corollary to the Triangle Sum Theorem**

$x = 30$ **Solve for *x*.**

▶ So, the measures of the acute angles are 30° and $2(30°) = 60°$.

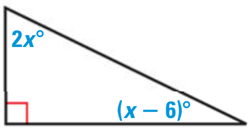
✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the measure of ∠1 in the diagram shown.



4. Find the measure of each interior angle of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$, and $m\angle C = 3x°$.

5. Find the measures of the acute angles of the right triangle in the diagram shown.



6. In Example 4, what is the measure of the obtuse angle formed between the staircase and a segment extending from the horizontal leg?

4.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 7, 20, 31, 43, and 51

SKILL PRACTICE

VOCABULARY **Match the triangle description with the most specific name.**

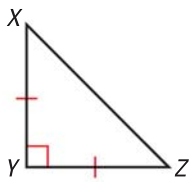
1. Angle measures: 30°, 60°, 90°	**A.** Isosceles
2. Side lengths: 2 cm, 2 cm, 2 cm	**B.** Scalene
3. Angle measures: 60°, 60°, 60°	**C.** Right
4. Side lengths: 6 m, 3 m, 6 m	**D.** Obtuse
5. Side lengths: 5 ft, 7 ft, 9 ft	**E.** Equilateral
6. Angle measures: 20°, 125°, 35°	**F.** Equiangular

7. ★ **WRITING** Can a right triangle also be obtuse? *Explain* why or why not.

EXAMPLE 1 on p. 217 for Exs. 8–10

CLASSIFYING TRIANGLES **Copy the triangle and measure its angles. Classify the triangle by its sides and by its angles.**

8.



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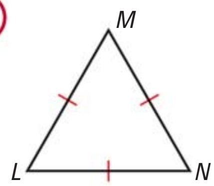

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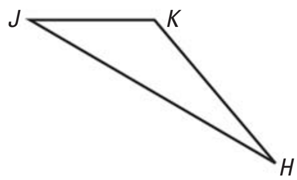


EXAMPLE 2 on p. 218 for Exs. 11–13

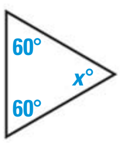
COORDINATE PLANE **A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle.**

11. $A(2, 3)$, $B(6, 3)$, $C(2, 7)$ **12.** $A(3, 3)$, $B(6, 9)$, $C(6, -3)$ **13.** $A(1, 9)$, $B(4, 8)$, $C(2, 5)$

EXAMPLE 3 on p. 219 for Exs. 14–19

FINDING ANGLE MEASURES **Find the value of x. Then classify the triangle by its angles.**

14.



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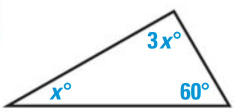

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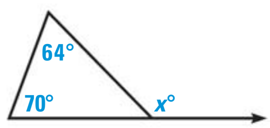
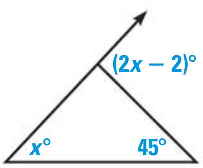
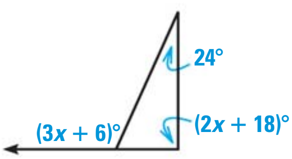


xy ALGEBRA **Find the measure of the exterior angle shown.**

17.



18.



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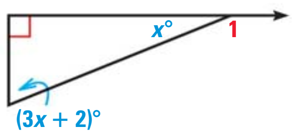
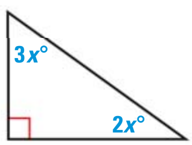


EXAMPLE 4 on p. 220 for Ex. 20

20. ★ **SHORT RESPONSE** *Explain* how to use the Corollary to the Triangle Sum Theorem to find the measure of each angle.

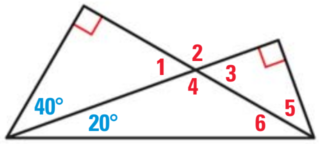


ANGLE RELATIONSHIPS **Find the measure of the numbered angle.**

21. $\angle 1$ **22.** $\angle 2$

23. $\angle 3$ **24.** $\angle 4$

25. $\angle 5$ **26.** $\angle 6$



27. **ALGEBRA** In $\triangle PQR$, $\angle P \cong \angle R$ and the measure of $\angle Q$ is twice the measure of $\angle R$. Find the measure of each angle.

28. **ALGEBRA** In $\triangle EFG$, $m\angle F = 3(m\angle G)$, and $m\angle E = m\angle F - 30°$. Find the measure of each angle.

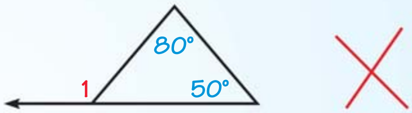
ERROR ANALYSIS **In Exercises 29 and 30, *describe* and correct the error.**

29.

All equilateral triangles are also isosceles. So, if $\triangle$ABC is isosceles, then it is equilateral as well.

30.

$m\angle 1 + 80° + 50° = 180°$



31. ★ **MULTIPLE CHOICE** Which of the following is not possible?

Ⓐ An acute scalene triangle

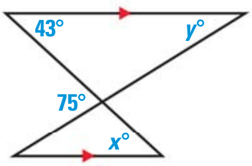
Ⓑ A triangle with two acute exterior angles

Ⓒ An obtuse isosceles triangle

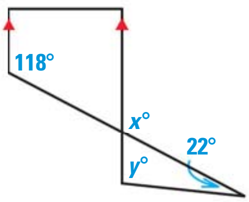
Ⓓ An equiangular acute triangle

ALGEBRA **In Exercises 32–37, find the values of *x* and *y*.**

32.



33.



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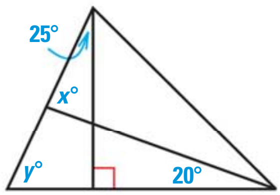

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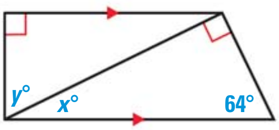

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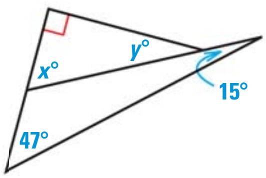

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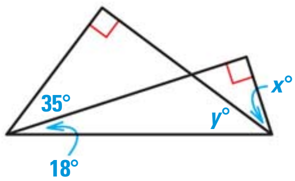


38. **VISUALIZATION** Is there an angle measure that is so small that any triangle with that angle measure will be an obtuse triangle? *Explain.*

39. **CHALLENGE** Suppose you have the equations $y = ax + b$, $y = cx + d$, and $y = ex + f$.

a. When will these three lines form a triangle?

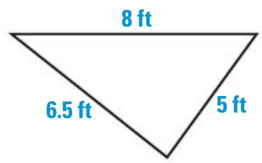
b. Let $c = 1$, $d = 2$, $e = 4$, and $f = -7$. Find values of a and b so that no triangle is formed by the three equations.

c. Draw the triangle formed when $a = \frac{4}{3}$, $b = \frac{1}{3}$, $c = -\frac{4}{3}$, $d = \frac{41}{3}$, $e = 0$, and $f = -1$. Then classify the triangle by its sides.

PROBLEM SOLVING

EXAMPLE 1 on p. 217 for Ex. 40

40. THEATER Three people are standing on a stage. The distances between the three people are shown in the diagram. Classify the triangle formed by its sides. Then copy the triangle, measure the angles, and classify the triangle by its angles.



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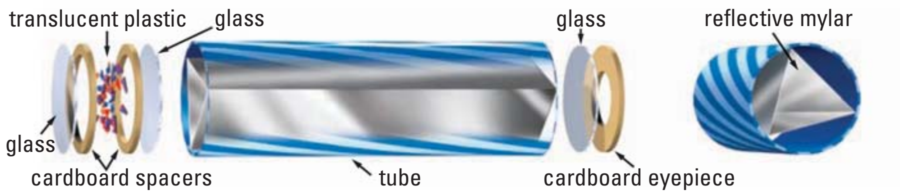
41. KALEIDOSCOPES You are making a kaleidoscope. The directions state that you are to arrange three pieces of reflective mylar in an equilateral and equiangular triangle. You must cut three strips from a piece of mylar 6 inches wide. What are the side lengths of the triangle used to form the kaleidoscope? What are the measures of the angles? *Explain*.



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42. SCULPTURE You are bending a strip of metal into an isosceles triangle for a sculpture. The strip of metal is 20 inches long. The first bend is made 6 inches from one end. *Describe* two ways you could complete the triangle.

43. ★ MULTIPLE CHOICE Which inequality describes the possible measures of an angle of a triangle?

Ⓐ $0° \le x° \le 180°$ Ⓑ $0° \le x° < 180°$ Ⓒ $0° < x° < 180°$ Ⓓ $0° < x° \le 180°$

SLING CHAIRS The brace of a sling chair forms a triangle with the seat and legs of the chair. Suppose $m\angle 2 = 50°$ and $m\angle 3 = 65°$.

44. Find $m\angle 6$. **45.** Find $m\angle 5$.

46. Find $m\angle 1$. **47.** Find $m\angle 4$.

48. PROOF Prove the Corollary to the Triangle Sum Theorem on page 220.

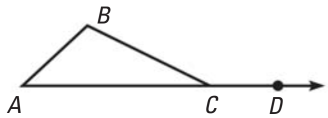
49. MULTI-STEP PROBLEM The measures of the angles of a triangle are $(2\sqrt{2x}°)$, $(5\sqrt{2x}°)$, and $(2\sqrt{2x}°)$.

a. Write an equation to show the relationship of the angles.

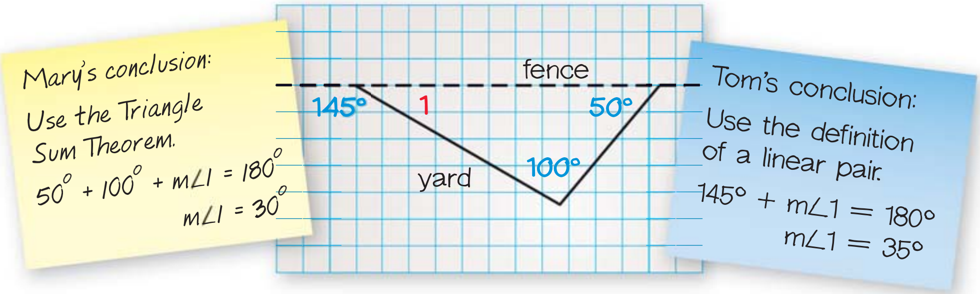
b. Find the measure of each angle.

c. Classify the triangle by its angles.

50. PROVING THEOREM 4.2 Prove the Exterior Angle Theorem. (*Hint:* Find two equations involving $m\angle ACB$.)



51. ★ **EXTENDED RESPONSE** The figure below shows an initial plan for a triangular flower bed that Mary and Tom plan to build along a fence. They are discussing what the measure of ∠1 should be.



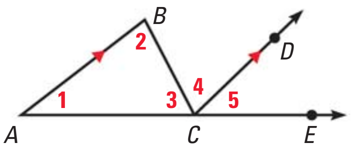
Did Mary and Tom both reason correctly? If not, who made a mistake and what mistake was made? If they did both reason correctly, what can you conclude about their initial plan? *Explain*.

52. **ALGEBRA** $\triangle ABC$ is isosceles. $AB = x$ and $BC = 2x - 4$.

a. Find two possible values for x if the perimeter of $\triangle ABC$ is 32.

b. How many possible values are there for x if the perimeter of $\triangle ABC$ is 12?

53. **CHALLENGE** Use the diagram to write a proof of the Triangle Sum Theorem. Your proof should be different than the proof of the Triangle Sum Theorem on page 219.



MIXED REVIEW

PREVIEW
Prepare for Lesson 4.2 in Exs. 57–59.

∠A and ∠B are complementary. Find m∠A and m∠B. *(p. 35)*

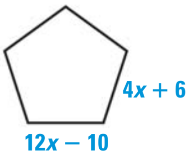
54. $m\angle A = (3x + 16)°$
$m\angle B = (4x - 3)°$

55. $m\angle A = (4x - 2)°$
$m\angle B = (7x + 4)°$

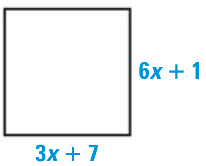
56. $m\angle A = (3x + 4)°$
$m\angle B = (2x + 6)°$

Each figure is a regular polygon. Find the value of x. *(p. 42)*

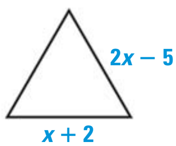
57.



58.



59.



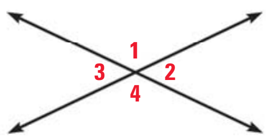
60. Use the Symmetric Property of Congruence to complete the statement: If __?__ ≅ __?__, then ∠*DEF* ≅ ∠*PQR*. *(p. 112)*

Use the diagram at the right. *(p. 124)*

61. If $m\angle 1 = 127°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

62. If $m\angle 4 = 170°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

63. If $m\angle 3 = 54°$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.



4.2 Apply Congruence and Triangles

Before You identified congruent angles.

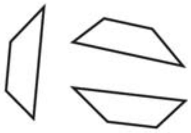
Now You will identify congruent figures.

Why? So you can determine if shapes are identical, as in Example 3.

Key Vocabulary
- **congruent figures**
- **corresponding parts**

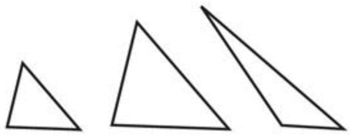
Two geometric figures are *congruent* if they have exactly the same size and shape. Imagine cutting out one of the congruent figures. You could then position the cut-out figure so that it fits perfectly onto the other figure.

Congruent



Same size and shape

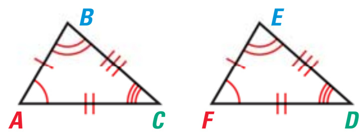
Not congruent



Different sizes or shapes

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

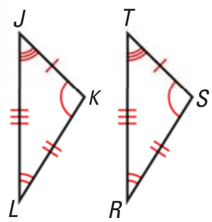
CONGRUENCE STATEMENTS When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles at the right are $\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.



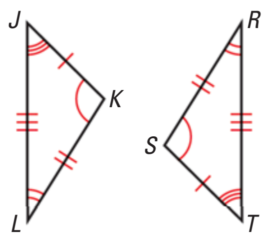
Corresponding angles	$\angle A \cong \angle F$	$\angle B \cong \angle E$	$\angle C \cong \angle D$
Corresponding sides	$\overline{AB} \cong \overline{FE}$	$\overline{BC} \cong \overline{ED}$	$\overline{AC} \cong \overline{FD}$

EXAMPLE 1 Identify congruent parts

VISUAL REASONING
To help you identify corresponding parts, turn $\triangle RST$.



Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

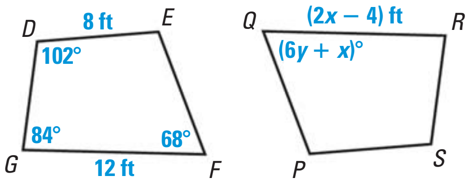
Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

EXAMPLE 2 Use properties of congruent figures

In the diagram, *DEFG* ≅ *SPQR*.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

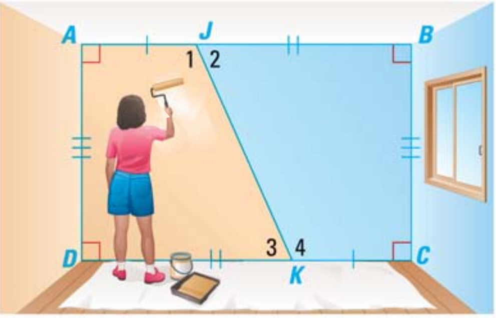
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Show that figures are congruent

PAINTING If you divide the wall into orange and blue sections along $\overline{JK}$, will the sections of the wall be the same size and shape? *Explain.*



Solution

From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

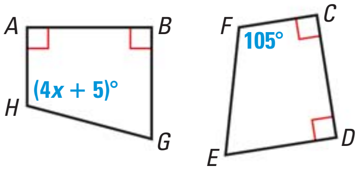
The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property, $\overline{JK} \cong \overline{KJ}$. All corresponding parts are congruent, so $AJKD \cong CKJB$.

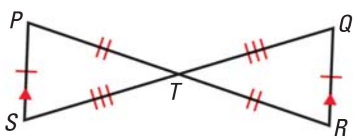
▶ Yes, the two sections will be the same size and shape.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In the diagram at the right, *ABGH* ≅ *CDEF*.

1. Identify all pairs of congruent corresponding parts.
2. Find the value of x and find $m\angle H$.
3. Show that $\triangle PTS \cong \triangle RTQ$.



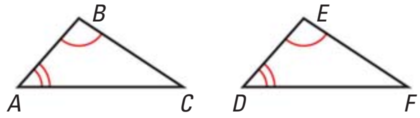


THEOREM *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

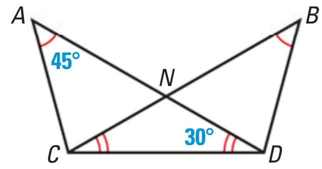
Proof: Ex. 28, p. 230



If $\angle A \cong \angle D$, and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle BDC$.



Solution

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

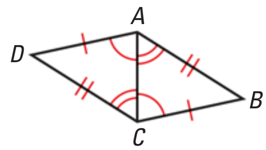
▶ So, $m\angle ACD = m\angle BDC = 105°$ by the definition of congruent angles.

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn to page 232 for the **Problem Solving Workshop**.

EXAMPLE 5 Prove that triangles are congruent

Write a proof.



GIVEN ▶ $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

PROVE ▶ $\triangle ACD \cong \triangle CAB$

Plan for Proof

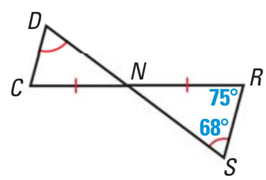
a. Use the Reflexive Property to show that $\overline{AC} \cong \overline{AC}$.

b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	1. Given
a.	2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property of Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	3. Given
b.	4. $\angle B \cong \angle D$	4. Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	5. Definition of $\cong$ $\triangle$s

✓ GUIDED PRACTICE for Examples 4 and 5



4. In the diagram, what is $m\angle DCN$?

5. By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?

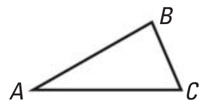
PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM *For Your Notebook*

THEOREM 4.4 Properties of Congruent Triangles

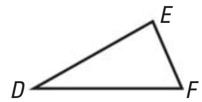
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$.



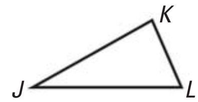
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.2 EXERCISES

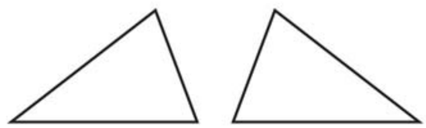
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.

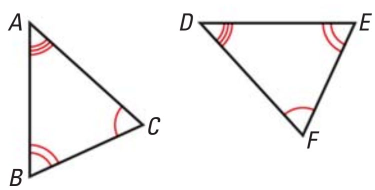


2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain.*

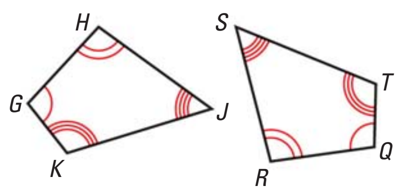
EXAMPLE 1 on p. 225 for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures.

3. $\triangle ABC \cong \triangle DEF$



4. $GHJK \cong QRST$



EXAMPLE 2 on p. 226 for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y =$ __?__

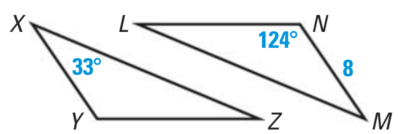
6. $m\angle M =$ __?__

7. $YX =$ __?__

8. $\overline{YZ} \cong$ __?__

9. $\triangle LNM \cong$ __?__

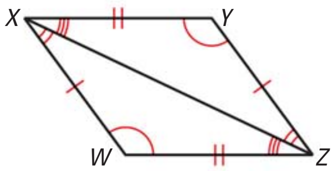
10. $\triangle YXZ \cong$ __?__



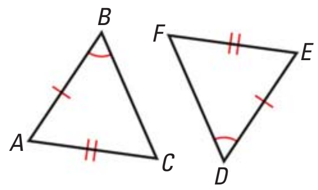
EXAMPLE 3
on p. 226
for Exs. 11–14

NAMING CONGRUENT FIGURES **Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.**

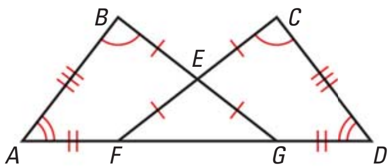
11.



12.



13.



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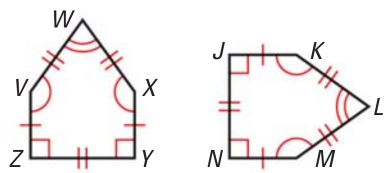
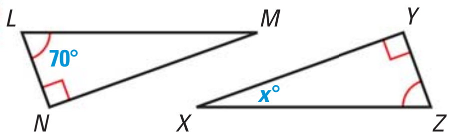


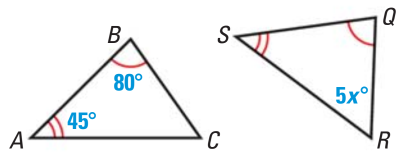
EXAMPLE 4
on p. 227
for Exs. 15–16

THIRD ANGLES THEOREM **Find the value of x.**

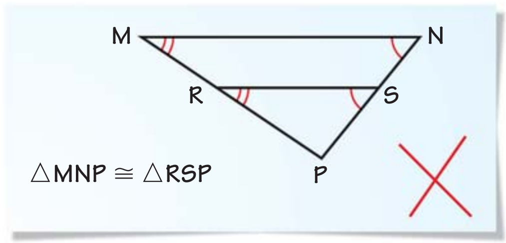
15.



16.



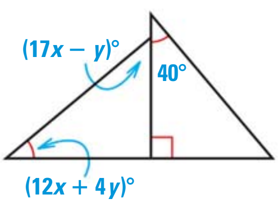
17. **ERROR ANALYSIS** A student says that $\triangle MNP \cong \triangle RSP$ because the corresponding angles of the triangles are congruent. *Describe* the error in this statement.



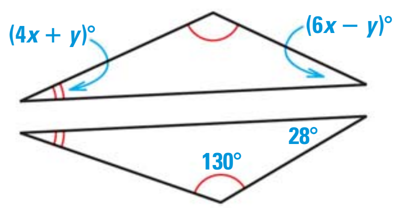
18. ★ **OPEN-ENDED MATH** Graph the triangle with vertices $L(3, 1)$, $M(8, 1)$, and $N(8, 8)$. Then graph a triangle congruent to $\triangle LMN$.

ALGEBRA **Find the values of x and y.**

19.



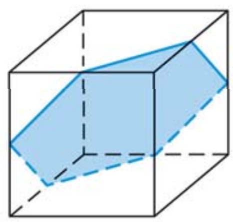
20.



21. ★ **MULTIPLE CHOICE** Suppose $\triangle ABC \cong \triangle EFD$, $\triangle EFD \cong \triangle GIH$, $m\angle A = 90°$, and $m\angle F = 20°$. What is $m\angle H$?

Ⓐ 20° Ⓑ 70° Ⓒ 90° Ⓓ Cannot be determined

22. **CHALLENGE** A hexagon is contained in a cube, as shown. Each vertex of the hexagon lies on the midpoint of an edge of the cube. This hexagon is equiangular. *Explain* why it is also regular.



PROBLEM SOLVING

23. RUG DESIGNS The rug design is made of congruent triangles. One triangular shape is used to make all of the triangles in the design. Which property guarantees that all the triangles are congruent?

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24. ★ OPEN-ENDED MATH Create a design for a rug made with congruent triangles that is different from the one in the photo above.

25. CAR STEREO A car stereo fits into a space in your dashboard. You want to buy a new car stereo, and it must fit in the existing space. What measurements need to be the same in order for the new stereo to be congruent to the old one?

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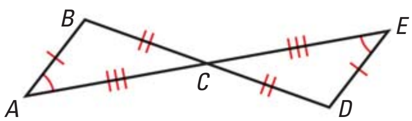
EXAMPLE 5
on p. 227
for Ex. 26

26. PROOF Copy and complete the proof.



GIVEN ▶ $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$

PROVE ▶ $\triangle ABC \cong \triangle EDC$

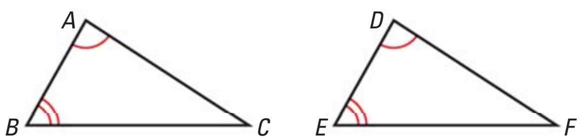
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$	1. Given
2. $\angle BCA \cong \angle DCE$	2. ?
3. ?	3. Third Angles Theorem
4. $\triangle ABC \cong \triangle EDC$	4. ?

27. ★ SHORT RESPONSE Suppose $\triangle ABC \cong \triangle DCB$, and the triangles share vertices at points *B* and *C*. Draw a figure that illustrates this situation. Is $\overline{AC} \parallel \overline{BD}$? *Explain.*

28. PROVING THEOREM 4.3 Use the plan to prove the Third Angles Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle B \cong \angle E$

PROVE ▶ $\angle C \cong \angle F$



Plan for Proof Use the Triangle Sum Theorem to show that the sums of the angle measures are equal. Then use substitution to show $\angle C \cong \angle F$.

○ = WORKED-OUT SOLUTIONS on p. WS1

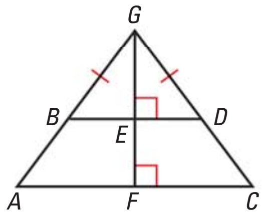
★ = STANDARDIZED TEST PRACTICE

29. **REASONING** Given that $\triangle AFC \cong \triangle DFE$, must F be the midpoint of $\overline{AD}$ and $\overline{EC}$? Include a drawing with your answer.

30. ★ **SHORT RESPONSE** You have a set of tiles that come in two different shapes, as shown. You can put two of the triangular tiles together to make a quadrilateral that is the same size and shape as the quadrilateral tile.

Explain how you can find all of the angle measures of each tile by measuring only two angles.

31. **MULTI-STEP PROBLEM** In the diagram, quadrilateral $ABEF \cong$ quadrilateral $CDEF$.



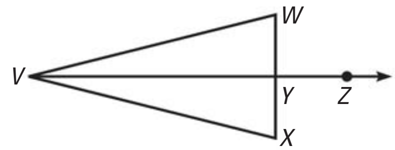
a. *Explain* how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

b. *Explain* how you know that $\angle GBE \cong \angle GDE$.

c. *Explain* how you know that $\angle GEB \cong \angle GED$.

d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? *Explain*.

32. **CHALLENGE** Use the diagram to write a proof.



GIVEN ▶ $\overline{WX} \perp \overrightarrow{VZ}$ at Y, Y is the midpoint of $\overline{WX}$, $\overline{VW} \cong \overline{VX}$, and $\overrightarrow{VZ}$ bisects $\angle WVX$.

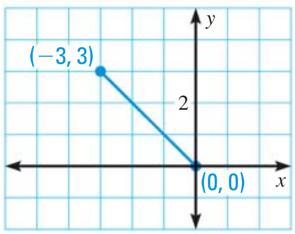
PROVE ▶ $\triangle VWY \cong \triangle VXY$

MIXED REVIEW

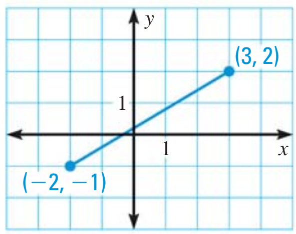
PREVIEW
Prepare for Lesson 4.3 in Exs. 33–35.

Use the Distance Formula to find the length of the segment. Round your answer to the nearest tenth of a unit. *(p. 15)*

33.



34.



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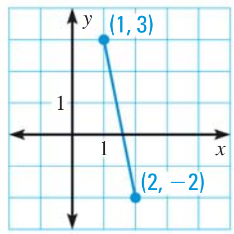
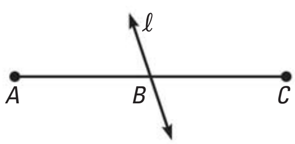
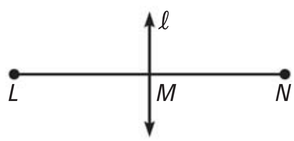


Line ℓ bisects the segment. Write a congruence statement. *(p. 15)*

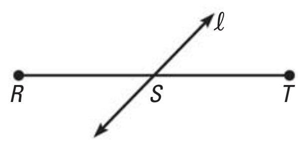
36.



37.



38.



Write the converse of the statement. *(p. 79)*

39. If three points are coplanar, then they lie in the same plane.

40. If the sky is cloudy, then it is raining outside.

PROBLEM SOLVING WORKSHOP
LESSON 4.2

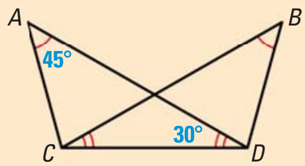
Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 227

MULTIPLE REPRESENTATIONS In Example 4 on page 227, you used congruencies in triangles that overlapped. When you solve problems like this, it may be helpful to redraw the art so that the triangles do not overlap.

PROBLEM

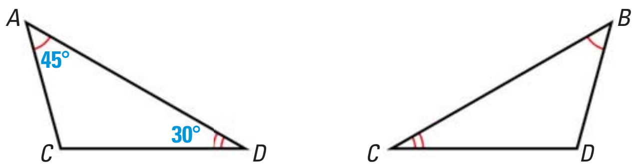
Find $m\angle BDC$.



METHOD

Drawing A Diagram

STEP 1 **Identify** the triangles that overlap. Then redraw them so that they are separate. Copy all labels and markings.



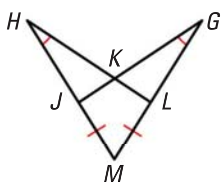
STEP 2 **Analyze** the situation. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

Also, because $\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, by the Third Angles Theorem, $\angle ACD \cong \angle BDC$, and $m\angle ACD = m\angle BDC = 105°$.

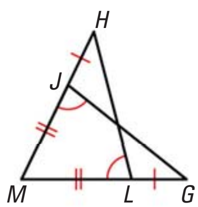
PRACTICE

1. **DRAWING FIGURES** Draw $\triangle HLM$ and $\triangle GJM$ so they do not overlap. Copy all labels and mark any known congruences.

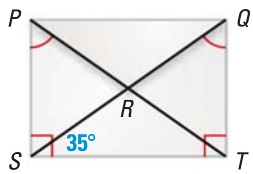
   a.



   b.



2. **ENVELOPE** Draw $\triangle PQS$ and $\triangle QPT$ so that they do not overlap. Find $m\angle PTS$.



Investigating Geometry ACTIVITY *Use before Lesson 4.3*

4.3 Investigate Congruent Figures

MATERIALS • straws • string • ruler • protractor

QUESTION How much information is needed to tell whether two figures are congruent?

EXPLORE 1 Compare triangles with congruent sides

STEP 1

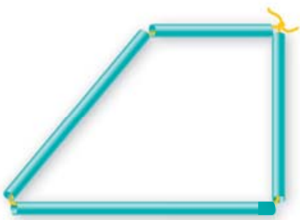
Make a triangle Cut straws to make side lengths of 8 cm, 10 cm, and 12 cm. Thread the string through the straws. Make a triangle by connecting the ends of the string.

STEP 2

Make another triangle Use the same length straws to make another triangle. If possible, make it different from the first. Compare the triangles. What do you notice?

EXPLORE 2 Compare quadrilaterals with congruent sides

STEP 1



Make a quadrilateral Cut straws to make side lengths of 5 cm, 7 cm, 9 cm, and 11 cm. Thread the string through the straws. Make a quadrilateral by connecting the string.

STEP 2

Make another quadrilateral Make a second quadrilateral using the same length straws. If possible, make it different from the first. Compare the quadrilaterals. What do you notice?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Can you make two triangles with the same side lengths that are different shapes? *Justify* your answer.
2. If you know that three sides of a triangle are congruent to three sides of another triangle, can you say the triangles are congruent? *Explain*.
3. Can you make two quadrilaterals with the same side lengths that are different shapes? *Justify* your answer.
4. If four sides of a quadrilateral are congruent to four sides of another quadrilateral, can you say the quadrilaterals are congruent? *Explain*.

4.3 Prove Triangles Congruent by SSS

Before You used the definition of congruent figures.

Now You will use the side lengths to prove triangles are congruent.

Why So you can determine if triangles in a tile floor are congruent, as in Ex. 22.

Key Vocabulary
- **congruent figures,** *p. 225*
- **corresponding parts,** *p. 225*

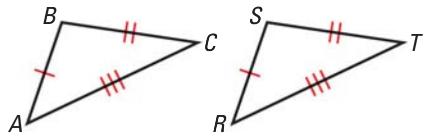
In the Activity on page 233, you saw that there is only one way to form a triangle given three side lengths. In general, any two triangles with the same three side lengths must be congruent.

POSTULATE — *For Your Notebook*

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{AB} \cong \overline{RS}$,
Side $\overline{BC} \cong \overline{ST}$, and
Side $\overline{CA} \cong \overline{TR}$,
then $\triangle ABC \cong \triangle RST$.

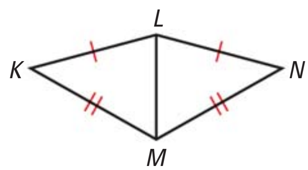


EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

PROVE ▶ $\triangle KLM \cong \triangle NLM$



Proof It is given that $\overline{KL} \cong \overline{NL}$ and $\overline{KM} \cong \overline{NM}$. By the Reflexive Property, $\overline{LM} \cong \overline{LM}$. So, by the SSS Congruence Postulate, $\triangle KLM \cong \triangle NLM$.

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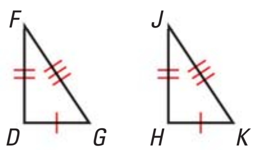
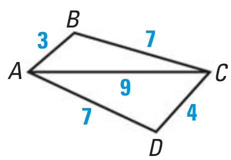
✓ GUIDED PRACTICE for Example 1

Decide whether the congruence statement is true. *Explain* your reasoning.

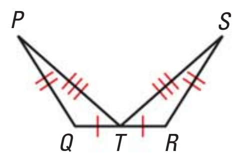
1. $\triangle DFG \cong \triangle HJK$



2. $\triangle ACB \cong \triangle CAD$

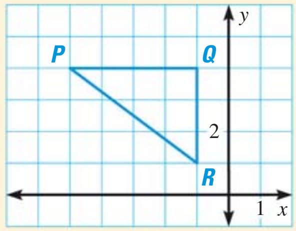


3. $\triangle QPT \cong \triangle RST$



★ EXAMPLE 2 Standardized Test Practice

Which are the coordinates of the vertices of a triangle congruent to $\triangle PQR$?



Ⓐ $(-1, 1), (-1, 5), (-4, 5)$

Ⓑ $(-2, 4), (-7, 4), (-4, 6)$

Ⓒ $(-3, 2), (-1, 3), (-3, 1)$

Ⓓ $(-7, 7), (-7, 9), (-3, 7)$

Solution

By counting, $PQ = 4$ and $QR = 3$. Use the Distance Formula to find PR.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PR = \sqrt{(-1 - (-5))^2 + (1 - 4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

By the SSS Congruence Postulate, any triangle with side lengths 3, 4, and 5 will be congruent to $\triangle PQR$. The distance from $(-1, 1)$ to $(-1, 5)$ is 4. The distance from $(-1, 5)$ to $(-4, 5)$ is 3. The distance from $(-1, 1)$ to $(-4, 5)$ is $\sqrt{(5 - 1)^2 + ((-4) - (-1))^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

ELIMINATE CHOICES

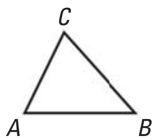
Once you know the side lengths of $\triangle PQR$, look for pairs of coordinates with the same x-coordinates or the same y-coordinates. In Choice C, $(-3, 2)$ and $(-3, 1)$ are only 1 unit apart. You can eliminate D in the same way.

✓ GUIDED PRACTICE for Example 2

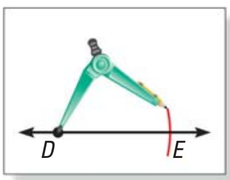
4. $\triangle JKL$ has vertices $J(-3, -2)$, $K(0, -2)$, and $L(-3, -8)$. $\triangle RST$ has vertices $R(10, 0)$, $S(10, -3)$, and $T(4, 0)$. Graph the triangles in the same coordinate plane and show that they are congruent.

ACTIVITY COPY A TRIANGLE

Follow the steps below to construct a triangle that is congruent to $\triangle ABC$.

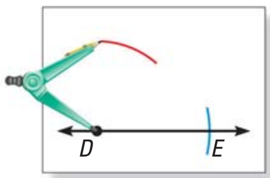


STEP 1



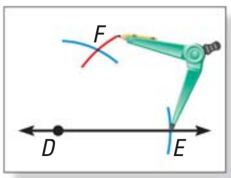
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

STEP 2



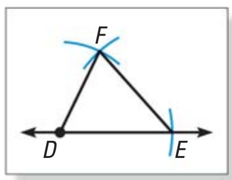
Open your compass to the length AC. Use this length to draw an arc with the compass point at D.

STEP 3



Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

STEP 4



Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 3 Solve a real-world problem

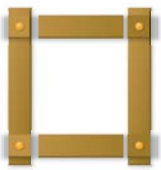
STRUCTURAL SUPPORT Explain why the bench with the diagonal support is stable, while the one without the support can collapse.

Solution

The bench with a diagonal support forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the bench is stable. The bench without a diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

✓ GUIDED PRACTICE for Example 3

Determine whether the figure is stable. *Explain* your reasoning.

5. 

6. 

7. 

4.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 16, 17, and 28

SKILL PRACTICE

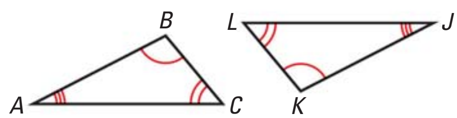
VOCABULARY **Tell whether the angles or sides are *corresponding angles*, *corresponding sides*, or *neither*.**

1. $\angle C$ and $\angle L$

2. $\overline{AC}$ and $\overline{JK}$

3. $\overline{BC}$ and $\overline{KL}$

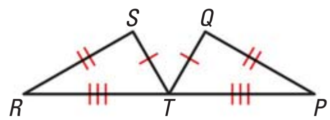
4. $\angle B$ and $\angle L$



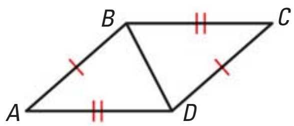
EXAMPLE 1 on p. 234 for Exs. 5–7

DETERMINING CONGRUENCE **Decide whether the congruence statement is true. *Explain* your reasoning.**

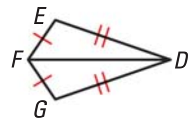
5. $\triangle RST \cong \triangle TQP$



6. $\triangle ABD \cong \triangle CDB$

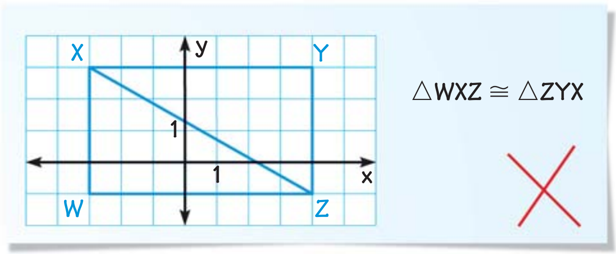


7. $\triangle DEF \cong \triangle DGF$



EXAMPLE 2
on p. 235
for Exs. 8–12

8. **ERROR ANALYSIS** *Describe* and correct the error in writing a congruence statement for the triangles in the coordinate plane.



ALGEBRA **Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.**

9. $A(-2, -2)$, $B(4, -2)$, $C(4, 6)$, $D(5, 7)$, $E(5, 1)$, $F(13, 1)$

10. $A(-2, 1)$, $B(3, -3)$, $C(7, 5)$, $D(3, 6)$, $E(8, 2)$, $F(10, 11)$

11. $A(0, 0)$, $B(6, 5)$, $C(9, 0)$, $D(0, -1)$, $E(6, -6)$, $F(9, -1)$

12. $A(-5, 7)$, $B(-5, 2)$, $C(0, 2)$, $D(0, 6)$, $E(0, 1)$, $F(4, 1)$

EXAMPLE 3
on p. 236
for Exs. 13–15

USING DIAGRAMS **Decide whether the figure is stable. *Explain*.**

13.


14.


15.

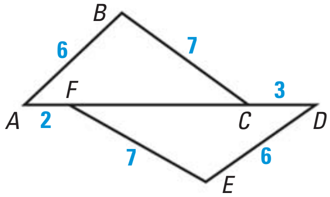

16. ★ **MULTIPLE CHOICE** Let $\triangle FGH$ be an equilateral triangle with point J as the midpoint of $\overline{FG}$. Which of the statements below is *not* true?

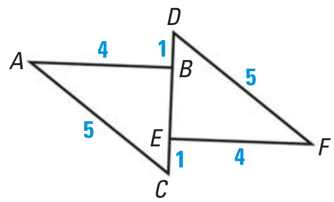
Ⓐ $\overline{FH} \cong \overline{GH}$ Ⓑ $\overline{FJ} \cong \overline{FH}$ Ⓒ $\overline{FJ} \cong \overline{GJ}$ Ⓓ $\triangle FHJ \cong \triangle GHJ$

17. ★ **MULTIPLE CHOICE** Let $ABCD$ be a rectangle separated into two triangles by $\overline{DB}$. Which of the statements below is *not* true?

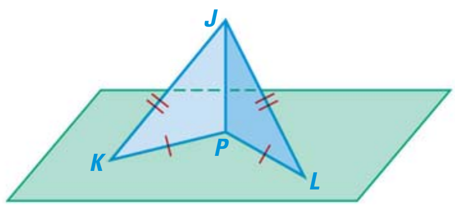
Ⓐ $\overline{AD} \cong \overline{CB}$ Ⓑ $\overline{AB} \cong \overline{AD}$ Ⓒ $\overline{AB} \cong \overline{CD}$ Ⓓ $\triangle DAB \cong \triangle BCD$

APPLYING SEGMENT ADDITION **Determine whether $\triangle ABC \cong \triangle DEF$. If they are congruent, write a congruence statement. *Explain* your reasoning.**

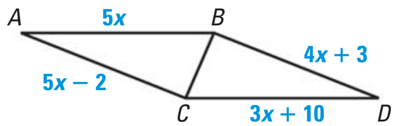
18.


19.


20. **3-D FIGURES** In the diagram, $\overline{PK} \cong \overline{PL}$ and $\overline{JK} \cong \overline{JL}$. Show that $\triangle JPK \cong \triangle JPL$.



21. **CHALLENGE** Find all values of x that make the triangles congruent. *Explain*.



PROBLEM SOLVING

EXAMPLE 1
on p. 234
for Ex. 22

22. TILE FLOORS You notice two triangles in the tile floor of a hotel lobby. You want to determine if the triangles are congruent, but you only have a piece of string. Can you determine if the triangles are congruent? *Explain*.

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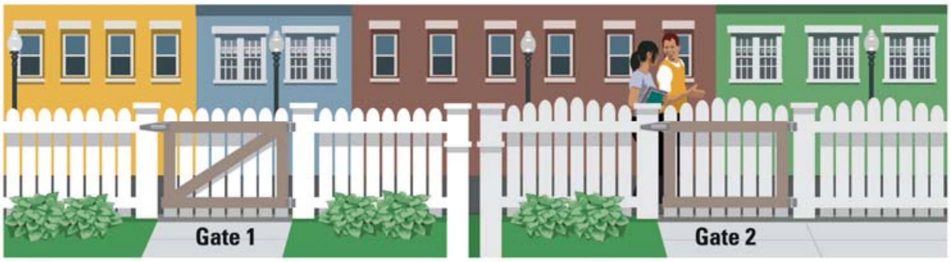
EXAMPLE 3
on p. 236
for Ex. 23

23. GATES Which gate is stable? *Explain* your reasoning.



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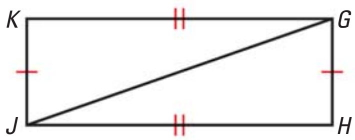
PROOF **Write a proof.**

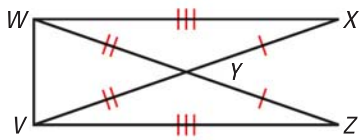
24. GIVEN ▶ $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$

PROVE ▶ $\triangle GHJ \cong \triangle JKG$



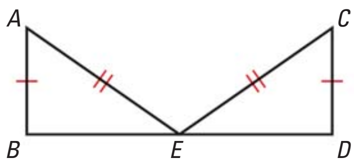
25. GIVEN ▶ $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$

PROVE ▶ $\triangle VWX \cong \triangle WVZ$



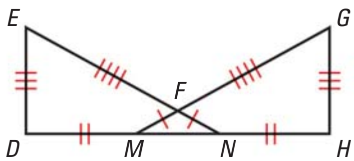
26. GIVEN ▶ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$, E is the midpoint of $\overline{BD}$.

PROVE ▶ $\triangle EAB \cong \triangle ECD$



27. GIVEN ▶ $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$

PROVE ▶ $\triangle DEN \cong \triangle HGM$



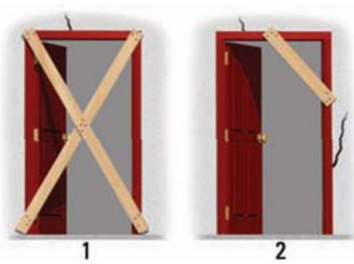
28. ★ EXTENDED RESPONSE When rescuers enter a partially collapsed building they often have to reinforce damaged doors for safety.

a. Diagonal braces are added to Door 1 as shown below. *Explain* why the door is more stable with the braces.

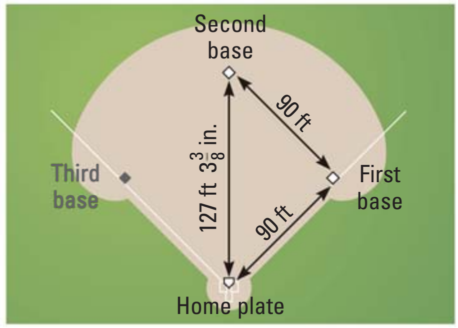
b. Would these braces be a good choice for rescuers needing to enter and exit the building through this doorway?

c. In the diagram, Door 2 has only a corner brace. Does this solve the problem from part (b)?

d. *Explain* why the corner brace makes the door more stable.



29. BASEBALL FIELD To create a baseball field, start by placing home plate. Then, place second base 127 feet $3\frac{3}{8}$ inches from home plate. Then, you can find first base using two tape measures. Stretch one from second base toward first base and the other from home plate toward first base. The point where the two tape measures cross at the 90 foot mark is first base. You can find third base in a similar manner. *Explain* how and why this process will always work.



30. CHALLENGE Draw and label the figure described below. Then, identify what is given and write a two-column proof.

In an isosceles triangle, if a segment is added from the vertex between the congruent sides to the midpoint of the third side, then two congruent triangles are formed.

MIXED REVIEW

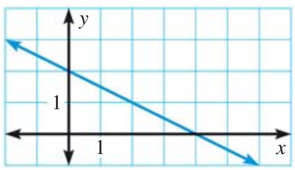
PREVIEW
Prepare for Lesson 4.4 in Exs. 31–33.

Find the slope of the line that passes through the points. *(p. 171)*

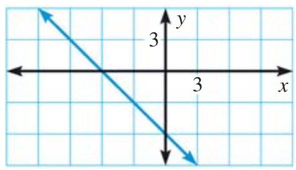
31. $A(3, 0), B(7, 4)$ **32.** $F(1, 8), G(-9, 2)$ **33.** $M(-4, -10), N(6, 2)$

Use the *x*- and *y*-intercepts to write an equation of the line. *(p. 180)*

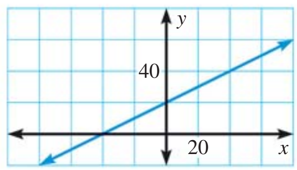
34.



35.



36.



37. Write an equation of a line that passes through $(-3, -1)$ and is parallel to $y = 3x + 2$. *(p. 180)*

QUIZ for Lessons 4.1–4.3

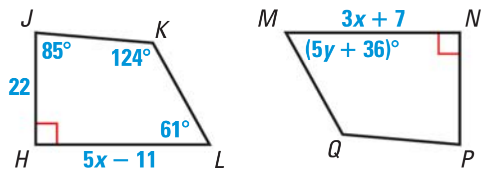
A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. *(p. 217)*

1. $A(-3, 0), B(0, 4), C(3, 0)$ **2.** $A(2, -4), B(5, -1), C(2, -1)$ **3.** $A(-7, 0), B(1, 6), C(-3, 4)$

In the diagram, $HJKL \cong NPQM$. *(p. 225)*

4. Find the value of x.

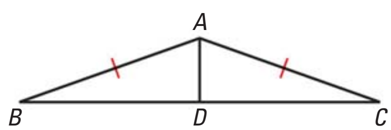
5. Find the value of y.



6. Write a proof. *(p. 234)*

GIVEN ▶ $\overline{AB} \cong \overline{AC}$, $\overline{AD}$ bisects $\overline{BC}$.

PROVE ▶ $\triangle ABD \cong \triangle ACD$



4.4 Prove Triangles Congruent by SAS and HL

Before	You used the SSS Congruence Postulate.
Now	You will use sides and angles to prove congruence.
Why?	So you can show triangles are congruent, as in Ex. 33.

Key Vocabulary
- **leg of a right triangle**
- **hypotenuse**

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.

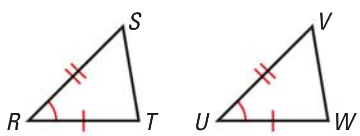
Any time you form an angle of the same measure with the pencils, the side formed by connecting the pencil points will have the same length. In fact, any two triangles formed in this way are congruent.

POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{RS} \cong \overline{UV}$,
Angle $\angle R \cong \angle U$, and
Side $\overline{RT} \cong \overline{UW}$,
then $\triangle RST \cong \triangle UVW$.

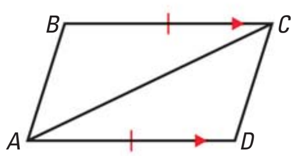


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$

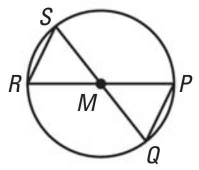


WRITE PROOFS
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

	STATEMENTS	REASONS
S	1. $\overline{BC} \cong \overline{DA}$	1. Given
	2. $\overline{BC} \parallel \overline{AD}$	2. Given
A	3. $\angle BCA \cong \angle DAC$	3. Alternate Interior Angles Theorem
S	4. $\overline{AC} \cong \overline{CA}$	4. Reflexive Property of Congruence
	5. $\triangle ABC \cong \triangle CDA$	5. SAS Congruence Postulate

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



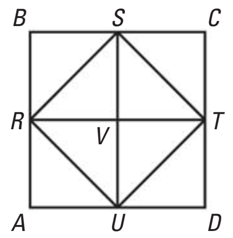
Solution

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so MP, MQ, MR, and MS are all equal.

▶ $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Postulate.

✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, $ABCD$ is a square with four congruent sides and four right angles. R, S, T, and U are the midpoints of the sides of $ABCD$. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.

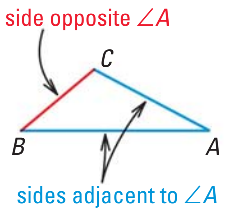


1. Prove that $\triangle SVR \cong \triangle UVR$.
2. Prove that $\triangle BSR \cong \triangle DUT$.

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

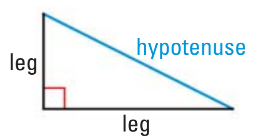
READ VOCABULARY

The two sides of a triangle that form an angle are *adjacent* to the angle. The side not adjacent to the angle is *opposite* the angle.



Therefore, SSA is *not* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

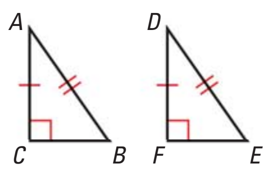
RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.



THEOREM *For Your Notebook*

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

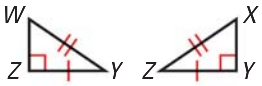


$\triangle ABC \cong \triangle DEF$

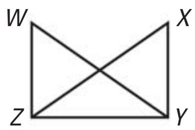
Proofs: Ex. 37, p. 439; p. 932

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

USE DIAGRAMS
If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



Write a proof.

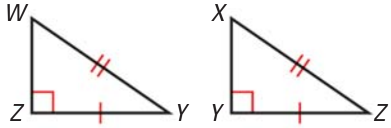


GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$

Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



STATEMENTS	REASONS
H **1.** $\overline{WY} \cong \overline{XZ}$	**1.** Given
2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	**2.** Given
3. $\angle Z$ and $\angle Y$ are right angles.	**3.** Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	**4.** Definition of a right triangle
L **5.** $\overline{ZY} \cong \overline{YZ}$	**5.** Reflexive Property of Congruence
6. $\triangle WYZ \cong \triangle XZY$	**6.** HL Congruence Theorem

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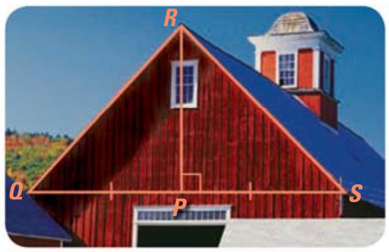
EXAMPLE 4 Choose a postulate or theorem

SIGN MAKING You are making a canvas sign to hang on the triangular wall over the door to the barn shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. What postulate or theorem can you use to conclude that $\triangle PQR \cong \triangle PSR$?



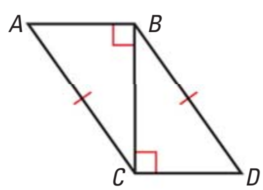
Solution

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property, $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two sides and their included angle are congruent.

▶ You can use the SAS Congruence Postulate to conclude that $\triangle PQR \cong \triangle PSR$.

✓ GUIDED PRACTICE for Examples 3 and 4

Use the diagram at the right.



3. Redraw $\triangle ACB$ and $\triangle DBC$ side by side with corresponding parts in the same position.

4. Use the information in the diagram to prove that $\triangle ACB \cong \triangle DBC$.

4.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 23, and 39

SKILL PRACTICE

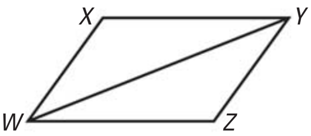
1. **VOCABULARY** Copy and complete: The angle between two sides of a triangle is called the __?__ angle.

2. ★ **WRITING** *Explain* the difference between proving triangles congruent using the SAS and SSS Congruence Postulates.

EXAMPLE 1 on p. 240 for Exs. 3–15

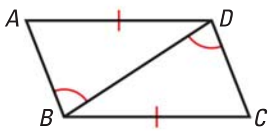
NAMING INCLUDED ANGLES **Use the diagram to name the included angle between the given pair of sides.**

3. $\overline{XY}$ and $\overline{YW}$
4. $\overline{WZ}$ and $\overline{ZY}$
5. $\overline{ZW}$ and $\overline{YW}$
6. $\overline{WX}$ and $\overline{YX}$
7. $\overline{XY}$ and $\overline{YZ}$
8. $\overline{WX}$ and $\overline{WZ}$

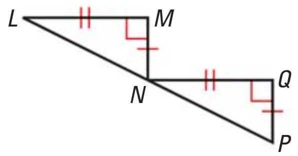


REASONING **Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.**

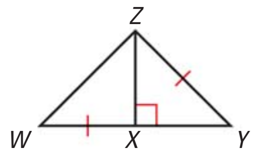
9. $\triangle ABD, \triangle CDB$



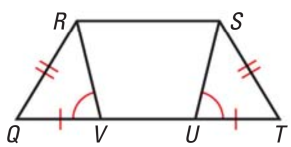
10. $\triangle LMN, \triangle NQP$



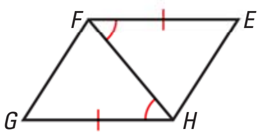
11. $\triangle YXZ, \triangle WXZ$



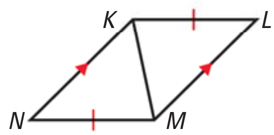
12. $\triangle QRV, \triangle TSU$



13. $\triangle EFH, \triangle GHF$



14. $\triangle KLM, \triangle MNK$



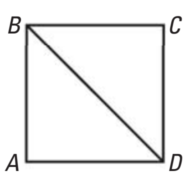
15. ★ **MULTIPLE CHOICE** Which of the following sets of information does not allow you to conclude that $\triangle ABC \cong \triangle DEF$?

   (A) $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \angle B \cong \angle E$
   (B) $\overline{AB} \cong \overline{DF}, \overline{AC} \cong \overline{DE}, \angle C \cong \angle E$
   (C) $\overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}, \overline{BA} \cong \overline{DE}$
   (D) $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \angle A \cong \angle D$

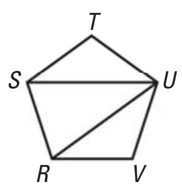
EXAMPLE 2 on p. 241 for Exs. 16–18

APPLYING SAS **In Exercises 16–18, use the given information to name two triangles that are congruent. *Explain* your reasoning.**

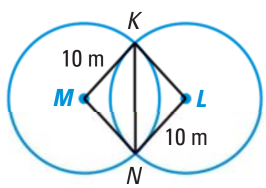
16. *ABCD* is a square with four congruent sides and four congruent angles.



17. *RSTUV* is a regular pentagon.

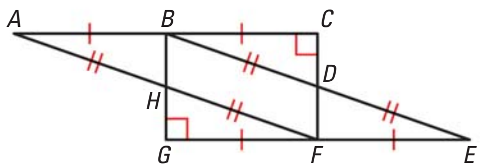


18. $\overline{MK} \perp \overline{MN}$ and $\overline{KL} \perp \overline{NL}$.



EXAMPLE 3
on p. 242
for Ex. 19

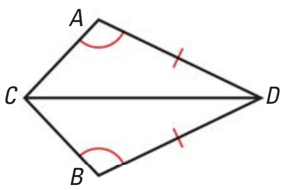
19. **OVERLAPPING TRIANGLES** Redraw $\triangle ACF$ and $\triangle EGB$ so they are side by side with corresponding parts in the same position. *Explain* how you know that $\triangle ACF \cong \triangle EGB$.



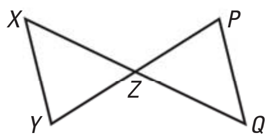
EXAMPLE 4
on p. 242
for Exs. 20–22

REASONING **Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.**

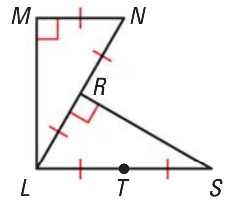
20.



21. Z is the midpoint of $\overline{PY}$ and $\overline{XQ}$.

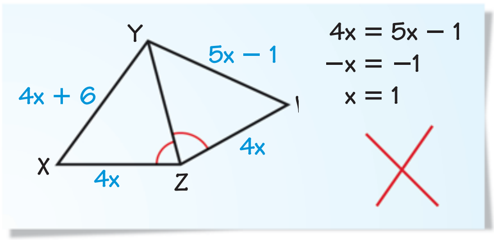


22.



23. ★ **WRITING** Suppose both pairs of corresponding legs of two right triangles are congruent. Are the triangles congruent? *Explain*.

24. **ERROR ANALYSIS** *Describe* and correct the error in finding the value of x.

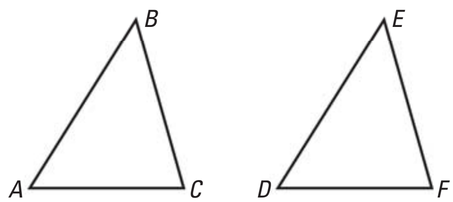


USING DIAGRAMS **In Exercises 25–27, state the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate.**

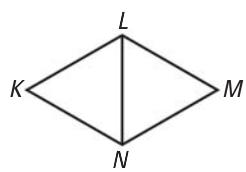
25. **GIVEN** ▶ $\overline{AB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, __?__ $\cong$ __?__
Use the SSS Congruence Postulate.

26. **GIVEN** ▶ $\angle A \cong \angle D$, $\overline{CA} \cong \overline{FD}$, __?__ $\cong$ __?__
Use the SAS Congruence Postulate.

27. **GIVEN** ▶ $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, __?__ $\cong$ __?__
Use the SAS Congruence Postulate.

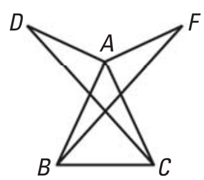


28. **USING ISOSCELES TRIANGLES** Suppose $\triangle KLN$ and $\triangle MLN$ are isosceles triangles with bases $\overline{KN}$ and $\overline{MN}$ respectively, and $\overline{NL}$ bisects $\angle KLM$. Is there enough information to prove that $\triangle KLN \cong \triangle MLN$? *Explain*.



29. **REASONING** Suppose M is the midpoint of $\overline{PQ}$ in $\triangle PQR$. If $\overline{RM} \perp \overline{PQ}$, *explain* why $\triangle RMP \cong \triangle RMQ$.

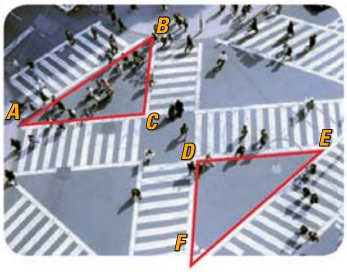
30. **CHALLENGE** Suppose $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AF}$, $\overline{AD} \perp \overline{AB}$, and $\overline{AF} \perp \overline{AC}$. *Explain* why you can conclude that $\triangle ACD \cong \triangle ABF$.

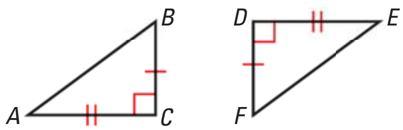


PROBLEM SOLVING

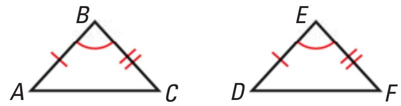
CONGRUENT TRIANGLES **In Exercises 31 and 32, identify the theorem or postulate you would use to prove the triangles congruent.**

31.





32.



33. SAILBOATS Suppose you have two sailboats. What information do you need to know to prove that the triangular sails are congruent using SAS? using HL?

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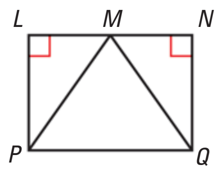
EXAMPLE 3
on p. 242
for Ex. 34

34. DEVELOPING PROOF Copy and complete the proof.



GIVEN ▶ Point M is the midpoint of $\overline{LN}$.
$\triangle PMQ$ is an isosceles triangle with base $\overline{PQ}$.
$\angle L$ and $\angle N$ are right angles.

PROVE ▶ $\triangle LMP \cong \triangle NMQ$

STATEMENTS	REASONS
1. $\angle L$ and $\angle N$ are right angles.	1. Given
2. $\triangle LMP$ and $\triangle NMQ$ are right triangles.	2. __?__
3. Point M is the midpoint of $\overline{LN}$.	3. __?__
4. __?__	4. Definition of midpoint
5. $\triangle PMQ$ is an isosceles triangle.	5. Given
6. __?__	6. Definition of isosceles triangle
7. $\triangle LMP \cong \triangle NMQ$	7. __?__

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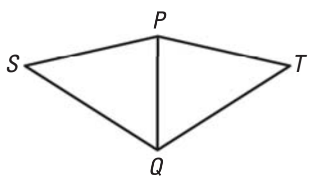
PROOF **In Exercises 35 and 36, write a proof.**

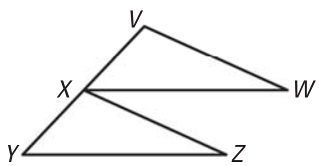
35. **GIVEN** ▶ $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

PROVE ▶ $\triangle SPQ \cong \triangle TPQ$



36. **GIVEN** ▶ $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$

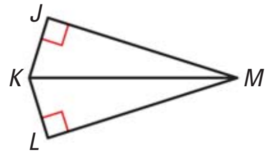
PROVE ▶ $\triangle VXW \cong \triangle XYZ$



PROOF **In Exercises 37 and 38, write a proof.**

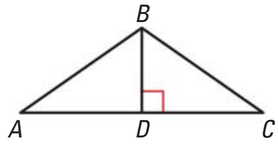
37. **GIVEN** ▶ $\overline{JM} \cong \overline{LM}$

PROVE ▶ $\triangle JKM \cong \triangle LKM$



38. **GIVEN** ▶ D is the midpoint of $\overline{AC}$.

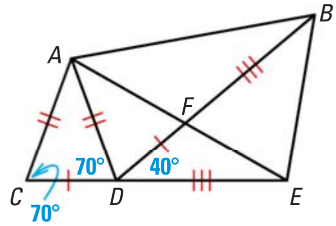
PROVE ▶ $\triangle ABD \cong \triangle CBD$



39. ★ **MULTIPLE CHOICE** Which triangle congruence can you prove, then use to prove that $\angle FED \cong \angle ABF$?

Ⓐ $\triangle ABE \cong \triangle ABF$ Ⓒ $\triangle AED \cong \triangle ABD$

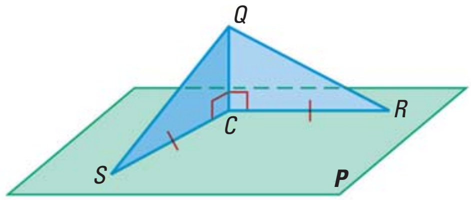
Ⓑ $\triangle ACD \cong \triangle ADF$ Ⓓ $\triangle AEC \cong \triangle ABD$



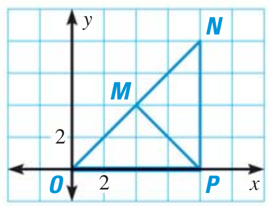
40. **PROOF** Write a two-column proof.

GIVEN ▶ $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$, $\overline{QC} \perp \overline{CS}$

PROVE ▶ $\triangle QCR \cong \triangle QCS$



41. **CHALLENGE** *Describe* how to show that $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate. Then show that the triangles are congruent using the SAS Congruence Postulate without measuring any angles. *Compare* the two methods.



MIXED REVIEW

Draw a figure that fits the description. *(p. 42)*

42. A pentagon that is not regular.

43. A quadrilateral that is equilateral but not equiangular.

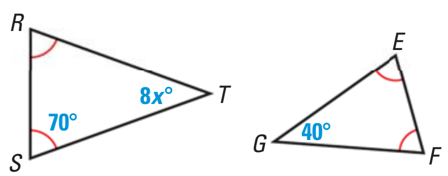
Write an equation of the line that passes through point *P* and is perpendicular to the line with the given equation. *(p. 180)*

44. $P(3, -1)$, $y = -x + 2$ **45.** $P(3, 3)$, $y = \frac{1}{3}x + 2$ **46.** $P(-4, -7)$, $y = -5$

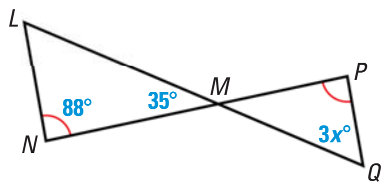
PREVIEW
Prepare for Lesson 4.5 in Exs. 47–48.

Find the value of *x*. *(p. 225)*

47.



48.



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4.4 Investigate Triangles and Congruence

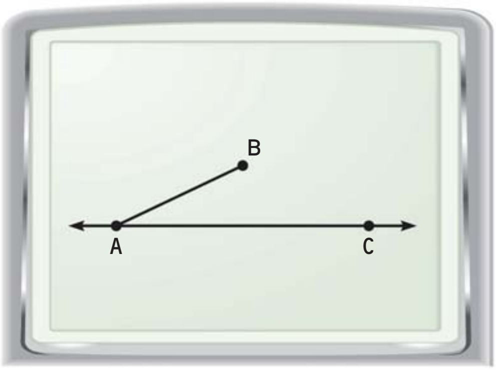
MATERIALS • graphing calculator or computer

QUESTION **Can you prove triangles are congruent by SSA?**

You can use geometry drawing software to show that if two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of another triangle, the triangles are not necessarily congruent.

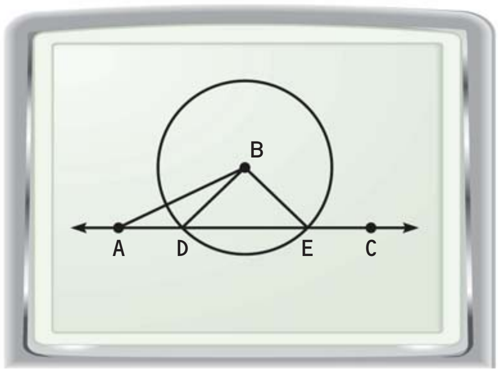
EXAMPLE **Draw two triangles**

STEP 1



Draw a line Draw points A and C. Draw line $\overleftrightarrow{AC}$. Then choose point B so that $\angle BAC$ is acute. Draw $\overline{AB}$.

STEP 2



Draw a circle Draw a circle with center at B so that the circle intersects $\overleftrightarrow{AC}$ at two points. Label the points D and E. Draw $\overline{BD}$ and $\overline{BE}$. Save as "EXAMPLE".

STEP 3 ***Use your drawing***

Explain why $\overline{BD} \cong \overline{BE}$. In $\triangle ABD$ and $\triangle ABE$, what other sides are congruent? What angles are congruent?

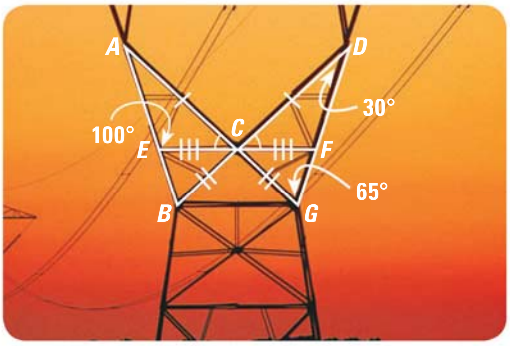
PRACTICE

1. *Explain* how your drawing shows that $\triangle ABD \not\cong \triangle ABE$.
2. Change the diameter of your circle so that it intersects $\overleftrightarrow{AC}$ in only one point. Measure $\angle BDA$. *Explain* why there is exactly one triangle you can draw with the measures AB, BD, and a 90° angle at $\angle BDA$.
3. *Explain* why your results show that SSA cannot be used to show that two triangles are congruent but that HL can.

MIXED REVIEW *of Problem Solving*

Lessons 4.1–4.4

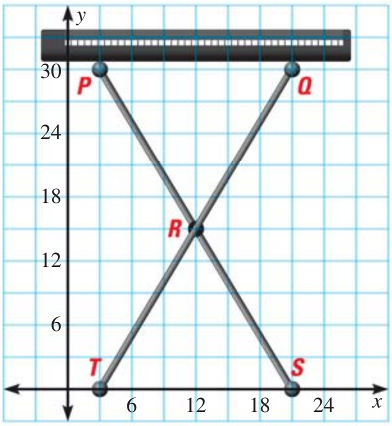
1. MULTI-STEP PROBLEM In the diagram, $\overline{AC} \cong \overline{CD}$, $\overline{BC} \cong \overline{CG}$, $\overline{EC} \cong \overline{CF}$, and $\angle ACE \cong \angle DCF$.



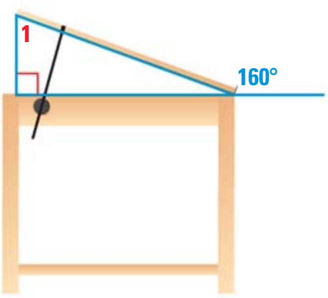
a. Classify each triangle in the figure by angles.

b. Classify each triangle in the figure by sides.

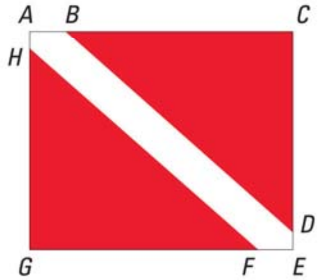
2. OPEN-ENDED *Explain* how you know that $\triangle PQR \cong \triangle STR$ in the keyboard stand shown.



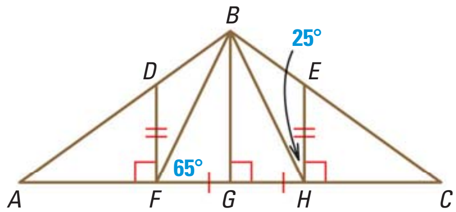
3. GRIDDED ANSWER In the diagram below, find the measure of $\angle 1$ in degrees.



4. SHORT RESPONSE A rectangular "diver down" flag is used to indicate that scuba divers are in the water. On the flag, $\overline{AB} \cong \overline{FE}$, $\overline{AH} \cong \overline{DE}$, $\overline{CE} \cong \overline{AG}$, and $\overline{EG} \cong \overline{AC}$. Also, $\angle A$, $\angle C$, $\angle E$, and $\angle G$ are right angles. Is $\triangle BCD \cong \triangle FGH$? *Explain*.



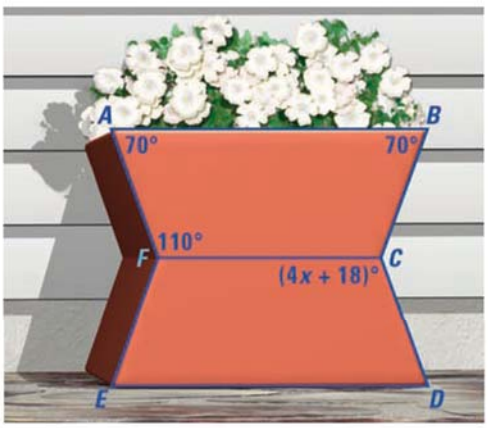
5. EXTENDED RESPONSE A roof truss is a network of pieces of wood that forms a stable structure to support a roof, as shown below.



a. Prove that $\triangle FGB \cong \triangle HGB$.

b. Is $\triangle BDF \cong \triangle BEH$? If so, prove it.

6. GRIDDED ANSWER In the diagram below, $BAFC \cong DEFC$. Find the value of x.



4.5 Prove Triangles Congruent by ASA and AAS

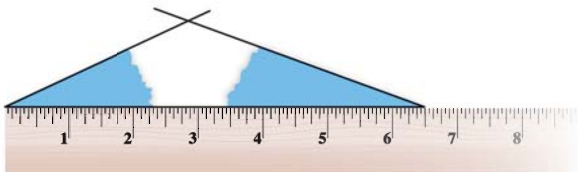
Before You used the SSS, SAS, and HL congruence methods.

Now You will use two more methods to prove congruences.

Why? So you can recognize congruent triangles in bikes, as in Exs. 23–24.

Key Vocabulary
- **flow proof**

Suppose you tear two angles out of a piece of paper and place them at a fixed distance on a ruler. Can you form more than one triangle with a given length and two given angle measures as shown below?



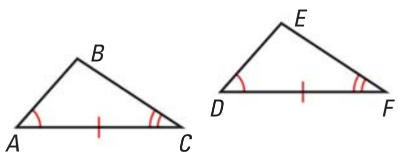
In a polygon, the side connecting the vertices of two angles is the *included* side. Given two angle measures and the length of the included side, you can make only one triangle. So, all triangles with those measurements are congruent.

THEOREMS — *For Your Notebook*

POSTULATE 21 Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

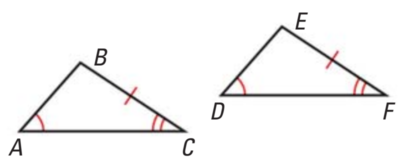
If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.6 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If **A**ngle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

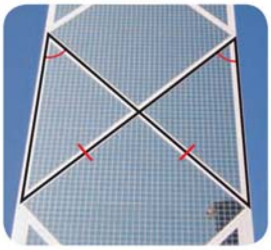


Proof: Example 2, p. 250

EXAMPLE 1 Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a.



b.

c.

Solution

a. The vertical angles are congruent, so two pairs of angles and a pair of non-included sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

b. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

c. Two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

AVOID ERRORS
You need at least one pair of congruent corresponding sides to prove two triangles congruent.

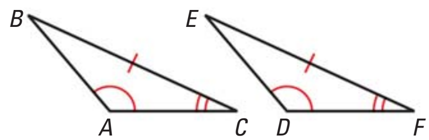
FLOW PROOFS You have written two-column proofs and paragraph proofs. A **flow proof** uses arrows to show the flow of a logical argument. Each reason is written below the statement it justifies.

EXAMPLE 2 Prove the AAS Congruence Theorem

Prove the Angle-Angle-Side Congruence Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ → $\angle B \cong \angle E$
Given

$\angle B \cong \angle E$ → $\triangle ABC \cong \triangle DEF$
Third ⦞ Thm.

$\angle C \cong \angle F$ → $\angle B \cong \angle E$; $\angle C \cong \angle F$ → $\triangle ABC \cong \triangle DEF$
Given

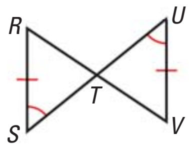
$\overline{BC} \cong \overline{EF}$ → $\triangle ABC \cong \triangle DEF$
Given

$\triangle ABC \cong \triangle DEF$
ASA Congruence Post.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

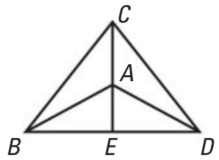
1. In the diagram at the right, what postulate or theorem can you use to prove that $\triangle RST \cong \triangle VUT$? *Explain.*



2. Rewrite the proof of the Triangle Sum Theorem on page 219 as a flow proof.

EXAMPLE 3 Write a flow proof

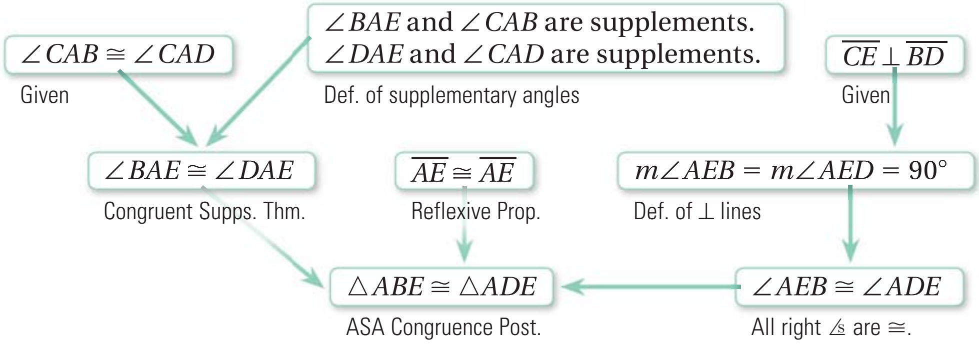
In the diagram, $\overline{CE} \perp \overline{BD}$ and $\angle CAB \cong \angle CAD$. Write a flow proof to show $\triangle ABE \cong \triangle ADE$.



Solution

GIVEN ▶ $\overline{CE} \perp \overline{BD}$, $\angle CAB \cong \angle CAD$

PROVE ▶ $\triangle ABE \cong \triangle ADE$

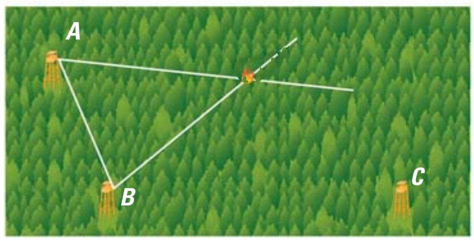


★ EXAMPLE 4 Standardized Test Practice

FIRE TOWERS The forestry service uses fire tower lookouts to watch for forest fires. When the lookouts spot a fire, they measure the angle of their view and radio a dispatcher. The dispatcher then uses the angles to locate the fire. How many lookouts are needed to locate a fire?

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ Not enough information

The locations of tower A, tower B, and the fire form a triangle. The dispatcher knows the distance from tower A to tower B and the measures of $\angle A$ and $\angle B$. So, he knows the measures of two angles and an included side of the triangle.



By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the fire location is given by the third vertex. Two lookouts are needed to locate the fire.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 3 and 4

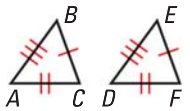
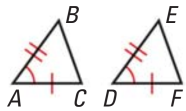
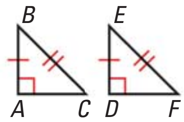
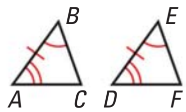
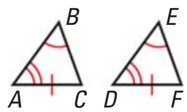
3. In Example 3, suppose $\angle ABE \cong \angle ADE$ is also given. What theorem or postulate besides ASA can you use to prove that $\triangle ABE \cong \triangle ADE$?

4. **WHAT IF?** In Example 4, suppose a fire occurs directly between tower B and tower C. Could towers B and C be used to locate the fire? *Explain.*

CONCEPT SUMMARY — *For Your Notebook*

Triangle Congruence Postulates and Theorems

You have learned five methods for proving that triangles are congruent.

SSS	SAS	HL (right △ only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **A**ngle-**L**eg, **L**eg-**L**eg, and **H**ypotenuse-**A**ngle.

4.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 21, and 26

SKILL PRACTICE

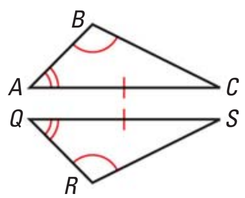
1. **VOCABULARY** Name one advantage of using a flow proof rather than a two-column proof.

2. ★ **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

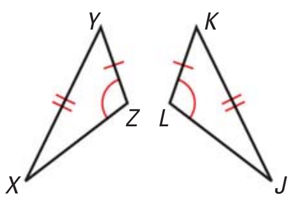
EXAMPLE 1 on p. 250 for Exs. 3–7

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.**

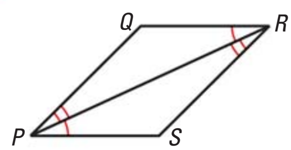
3. $\triangle ABC, \triangle QRS$



4. $\triangle XYZ, \triangle JKL$

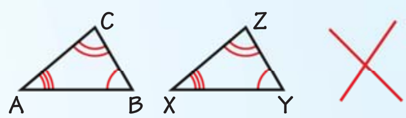


5. $\triangle PQR, \triangle RSP$



6. **ERROR ANALYSIS** *Describe* the error in concluding that $\triangle ABC \cong \triangle XYZ$.

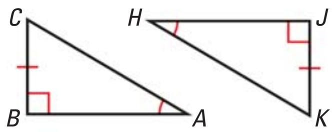
By AAA,
△ABC ≅ △XYZ.



7. ★ **MULTIPLE CHOICE** Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle HJK$?

Ⓐ ASA　　Ⓑ AAS

Ⓒ SAS　　Ⓓ Not enough information



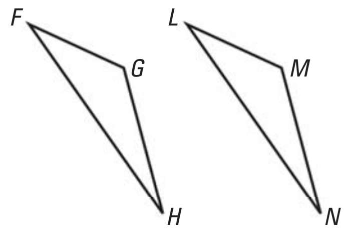
EXAMPLE 2
on p. 250
for Exs. 8–13

DEVELOPING PROOF **State the third congruence that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given postulate or theorem.**

8. **GIVEN** ▶ $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, ___?___ $\cong$ ___?___
Use the AAS Congruence Theorem.

9. **GIVEN** ▶ $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, ___?___ $\cong$ ___?___
Use the ASA Congruence Postulate.

10. **GIVEN** ▶ $\overline{FH} \cong \overline{LN}$, $\angle H \cong \angle N$, ___?___ $\cong$ ___?___
Use the SAS Congruence Postulate.

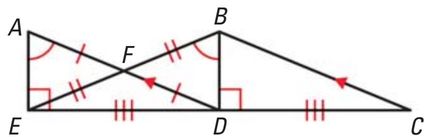


OVERLAPPING TRIANGLES ***Explain*** **how you can prove that the indicated triangles are congruent using the given postulate or theorem.**

11. $\triangle AFE \cong \triangle DFB$ by SAS

12. $\triangle AED \cong \triangle BDE$ by AAS

13. $\triangle AED \cong \triangle BDC$ by ASA



DETERMINING CONGRUENCE **Tell whether you can use the given information to determine whether $\triangle ABC \cong \triangle DEF$. *Explain* your reasoning.**

14. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

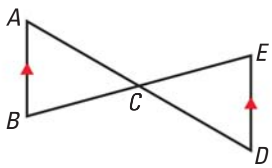
15. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

16. $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DE}$

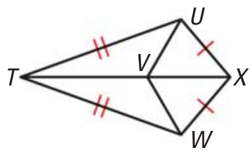
17. $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FD}$, $\overline{AC} \cong \overline{DE}$

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate(s) or theorem(s) you would use.**

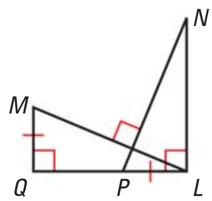
18. $\triangle ABC$, $\triangle DEC$



19. $\triangle TUV$, $\triangle TWV$



20. $\triangle QML$, $\triangle LPN$

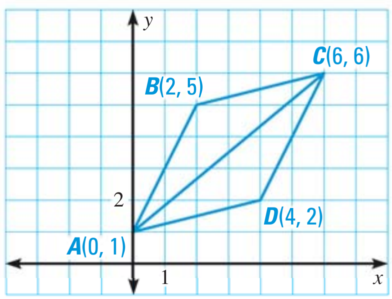


21. ★ **EXTENDED RESPONSE** Use the graph at the right.

  a. Show that $\angle CAD \cong \angle ACB$. *Explain* your reasoning.

  b. Show that $\angle ACD \cong \angle CAB$. *Explain* your reasoning.

  c. Show that $\triangle ABC \cong \triangle CDA$. *Explain* your reasoning.



22. **CHALLENGE** Use a coordinate plane.

  a. Graph the lines $y = 2x + 5$, $y = 2x - 3$, and $x = 0$ in the same coordinate plane.

  b. Consider the equation $y = mx + 1$. For what values of m will the graph of the equation form two triangles if added to your graph? For what values of m will those triangles be congruent? *Explain.*

PROBLEM SOLVING

CONGRUENCE IN BICYCLES ***Explain*** **why the triangles are congruent.**

23.

24.

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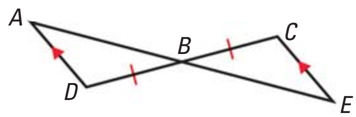
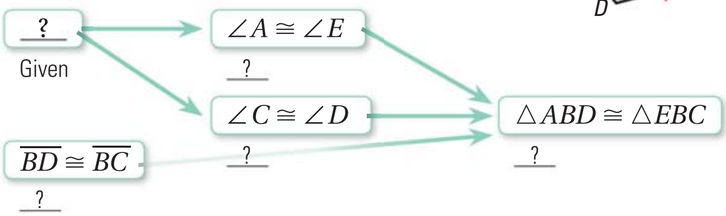
EXAMPLE 3 on p. 251 for Ex. 25

25. FLOW PROOF Copy and complete the flow proof.

GIVEN ▶ $\overline{AD} \parallel \overline{CE}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$





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EXAMPLE 4 on p. 251 for Ex. 26

26. ★ **SHORT RESPONSE** You are making a map for an orienteering race. Participants start at a large oak tree, find a boulder 250 yards due east of the oak tree, and then find a maple tree that is 50° west of north of the boulder and 35° east of north of the oak tree. Sketch a map. Can you locate the maple tree? *Explain*.

27. AIRPLANE In the airplane at the right, $\angle C$ and $\angle F$ are right angles, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$. What postulate or theorem allows you to conclude that $\triangle ABC \cong \triangle DEF$?

RIGHT TRIANGLES **In Lesson 4.4, you learned the Hypotenuse-Leg Theorem for right triangles. In Exercises 28–30, write a paragraph proof for these other theorems about right triangles.**

28. Leg-Leg (LL) Theorem If the legs of two right triangles are congruent, then the triangles are congruent.

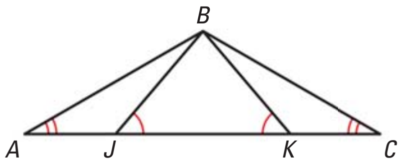
29. Angle-Leg (AL) Theorem If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.

30. Hypotenuse-Angle (HA) Theorem If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent.

31. PROOF Write a two-column proof.

GIVEN ▶ $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

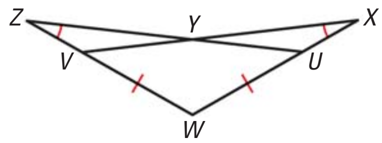
PROVE ▶ $\triangle ABK \cong \triangle CBJ$



32. PROOF Write a flow proof.

GIVEN ▶ $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

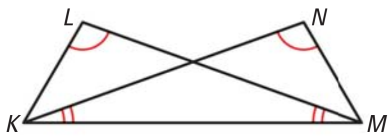
PROVE ▶ $\triangle XWV \cong \triangle ZWU$



33. PROOF Write a proof.

GIVEN ▶ $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

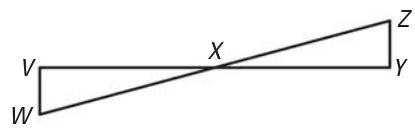
PROVE ▶ $\triangle NMK \cong \triangle LKM$



34. PROOF Write a proof.

GIVEN ▶ X is the midpoint of $\overline{VY}$ and $\overline{WZ}$.

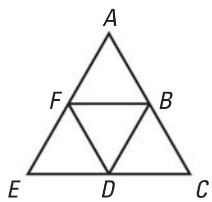
PROVE ▶ $\triangle VWX \cong \triangle YZX$



35. CHALLENGE Write a proof.

GIVEN ▶ $\triangle ABF \cong \triangle DFB$, F is the midpoint of $\overline{AE}$, B is the midpoint of $\overline{AC}$.

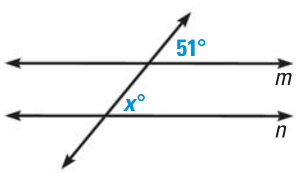
PROVE ▶ $\triangle FDE \cong \triangle BCD \cong \triangle ABF$



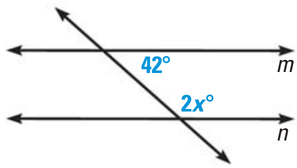
MIXED REVIEW

Find the value of x that makes $m \parallel n$. *(p. 161)*

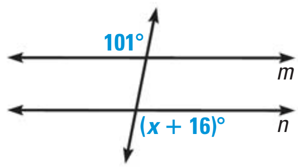
36.



37.



38.



Write an equation of the line that passes through point P and is parallel to the line with the given equation. *(p. 180)*

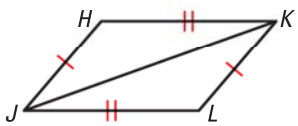
39. $P(0, 3)$, $y = x - 8$

40. $P(-2, 4)$, $y = -2x + 3$

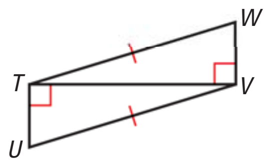
PREVIEW
Prepare for Lesson 4.6 in Exs. 41–43.

Decide which method, SSS, SAS, or HL, can be used to prove that the triangles are congruent. *(pp. 234, 240)*

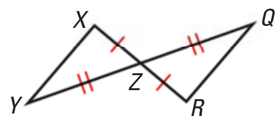
41. $\triangle HJK \cong \triangle LKJ$



42. $\triangle UTV \cong \triangle WVT$



43. $\triangle XYZ \cong \triangle RQZ$



4.6 Use Congruent Triangles

Before You used corresponding parts to prove triangles congruent.

Now You will use congruent triangles to prove corresponding parts congruent.

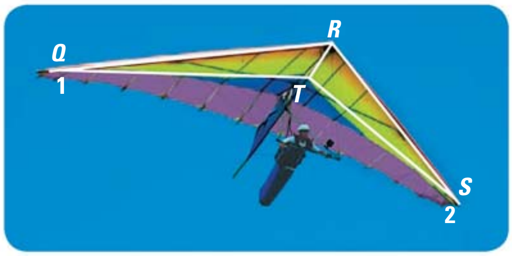
Why? So you can find the distance across a half pipe, as in Ex. 30.

Key Vocabulary
- **corresponding parts,** *p. 225*

By definition, congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, you know that their corresponding parts must be congruent as well.

EXAMPLE 1 Use congruent triangles

Explain how you can use the given information to prove that the hanglider parts are congruent.



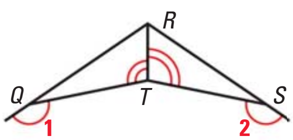
GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$

PROVE ▶ $\overline{QT} \cong \overline{ST}$

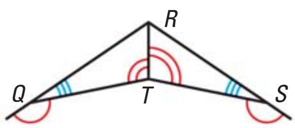
Solution

If you can show that $\triangle QRT \cong \triangle SRT$, you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$.

Mark given information.



Add deduced information.



Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle QRT \cong \triangle SRT$. Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

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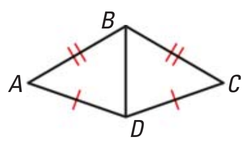
✓ GUIDED PRACTICE for Example 1

1. *Explain* how you can prove that $\angle A \cong \angle C$.

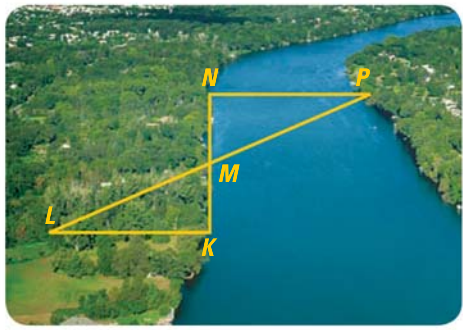


EXAMPLE 2 Use congruent triangles for measurement

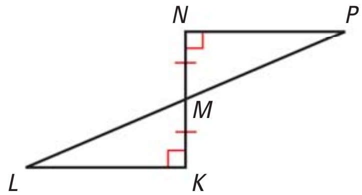
INDIRECT MEASUREMENT

When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

SURVEYING Use the following method to find the distance across a river, from point N to point P.



- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M, the midpoint of $\overline{NK}$.
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L, P, and M are collinear.
- Explain how this plan allows you to find the distance.

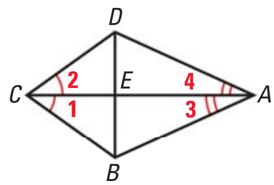


Solution

Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance NP across the river by measuring $\overline{KL}$.

EXAMPLE 3 Plan a proof involving pairs of triangles

Use the given information to write a plan for proof.



GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\triangle BCE \cong \triangle DCE$

Solution

In $\triangle BCE$ and $\triangle DCE$, you know $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property. You can use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$.

▶ **Plan for Proof** Use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle BCE \cong \triangle DCE$.

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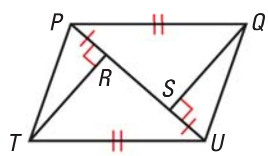
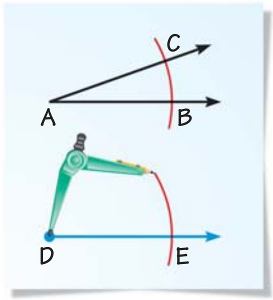
✓ GUIDED PRACTICE for Examples 2 and 3



2. In Example 2, does it matter how far from point N you place a stake at point K? *Explain.*
3. Using the information in the diagram at the right, write a plan to prove that $\triangle PTU \cong \triangle UQP$.

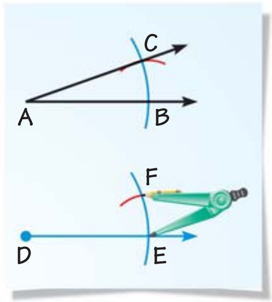
PROVING CONSTRUCTIONS On page 34, you learned how to use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

STEP 1



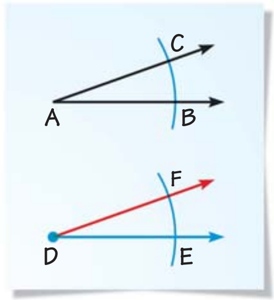
To copy $\angle A$, draw a segment with initial point D. Draw an arc with center A. Using the same radius, draw an arc with center D. Label points B, C, and E.

STEP 2



Draw an arc with radius BC and center E. Label the intersection F.

STEP 3



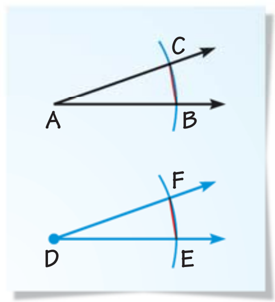
Draw $\overrightarrow{DF}$. In Example 4, you will prove that $\angle D \cong \angle A$.

EXAMPLE 4 Prove a construction

Write a proof to verify that the construction for copying an angle is valid.

Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$ are all determined by the same compass setting, as are $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.



GIVEN ▶ $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$

PROVE ▶ $\angle D \cong \angle A$

Plan for Proof Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle A$ and $\angle D$ are congruent.

Plan in Action

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$	1. Given
2. $\triangle FDE \cong \triangle CAB$	2. SSS Congruence Postulate
3. $\angle D \cong \angle A$	3. Corresp. parts of $\cong$ $\triangle$s are $\cong$.

✓ GUIDED PRACTICE for Example 4

4. Look back at the construction of an angle bisector in Explore 4 on page 34. What segments can you assume are congruent?

4.6 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 19, 23, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 31, and 36

SKILL PRACTICE

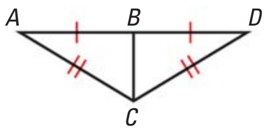
1. **VOCABULARY** Copy and complete: Corresponding parts of congruent triangles are __?__.

2. ★ **WRITING** *Explain* why you might choose to use congruent triangles to measure the distance across a river. Give another example where it may be easier to measure with congruent triangles rather than directly.

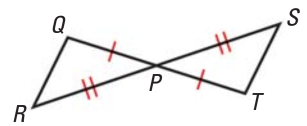
EXAMPLES 1 and 2 on p. 256–257 for Exs. 3–11

CONGRUENT TRIANGLES **Tell which triangles you can show are congruent in order to prove the statement. What postulate or theorem would you use?**

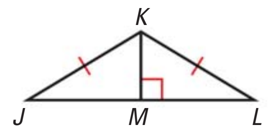
3. $\angle A \cong \angle D$



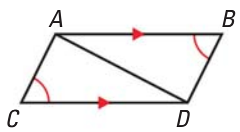
4. $\angle Q \cong \angle T$



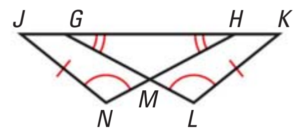
5. $\overline{JM} \cong \overline{LM}$



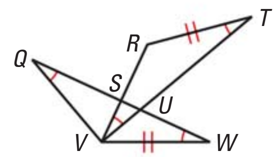
6. $\overline{AC} \cong \overline{BD}$



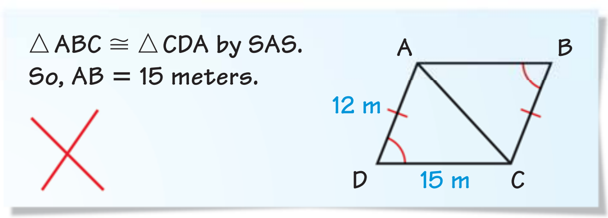
7. $\overline{GK} \cong \overline{HJ}$



8. $\overline{QW} \cong \overline{TV}$

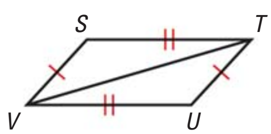


9. **ERROR ANALYSIS** *Describe* the error in the statement.

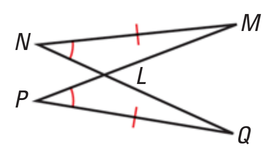


PLANNING FOR PROOF **Use the diagram to write a plan for proof.**

10. **PROVE** ▶ $\angle S \cong \angle U$



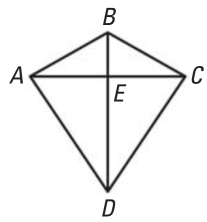
11. **PROVE** ▶ $\overline{LM} \cong \overline{LQ}$



12. **PENTAGONS** *Explain* why segments connecting any pair of corresponding vertices of congruent pentagons are congruent. Make a sketch to support your answer.

13. *xy* **ALGEBRA** Given that $\triangle ABC \cong \triangle DEF$, $m\angle A = 70°$, $m\angle B = 60°$, $m\angle C = 50°$, $m\angle D = (3x + 10)°$, $m\angle E = \left(\frac{y}{3} + 20\right)°$, and $m\angle F = (z^2 + 14)°$, find the values of x, y, and z.

14. ★ **MULTIPLE CHOICE** Which set of given information does *not* allow you to conclude that $\overline{AD} \cong \overline{CD}$?



Ⓐ $\overline{AE} \cong \overline{CE}$, $m\angle BEA = 90°$

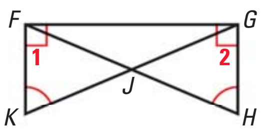
Ⓑ $\overline{BA} \cong \overline{BC}$, $\angle BDC \cong \angle BDA$

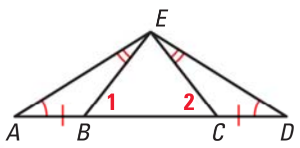
Ⓒ $\overline{AB} \cong \overline{CB}$, $\angle ABE \cong \angle CBE$

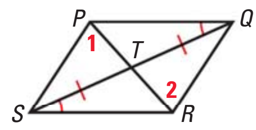
Ⓓ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CB}$

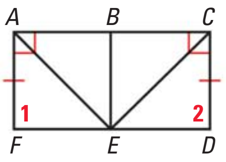
EXAMPLE 3
on p. 257
for Exs. 15–20

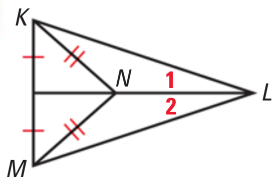
PLANNING FOR PROOF **Use the information given in the diagram to write a plan for proving that $\angle 1 \cong \angle 2$.**

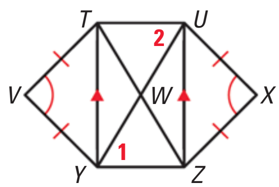
15.


16.


17.


18.


19.


20.


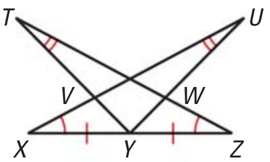
USING COORDINATES **Use the vertices of $\triangle ABC$ and $\triangle DEF$ to show that $\angle A \cong \angle D$. *Explain* your reasoning.**

21. $A(3, 7)$, $B(6, 11)$, $C(11, 13)$, $D(2, -4)$, $E(5, -8)$, $F(10, -10)$

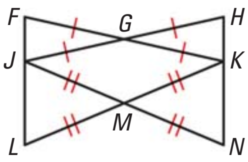
22. $A(3, 8)$, $B(3, 2)$, $C(11, 2)$, $D(-1, 5)$, $E(5, 5)$, $F(5, 13)$

PROOF **Use the information given in the diagram to write a proof.**

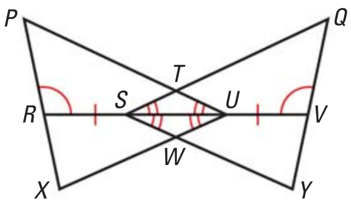
23. **PROVE ▶** $\angle VYX \cong \angle WYZ$



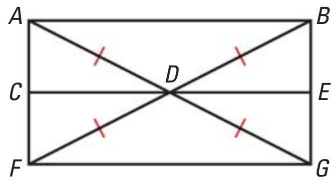
24. **PROVE ▶** $\overline{FL} \cong \overline{HN}$



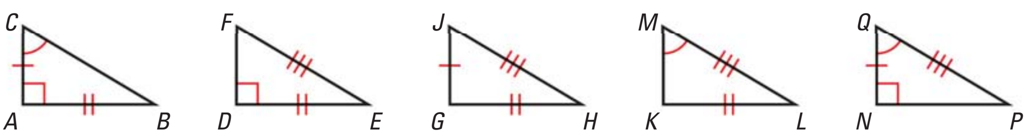
25. **PROVE ▶** $\triangle PUX \cong \triangle QSY$



26. **PROVE ▶** $\overline{AC} \cong \overline{GE}$



27. **CHALLENGE** Which of the triangles below are congruent?



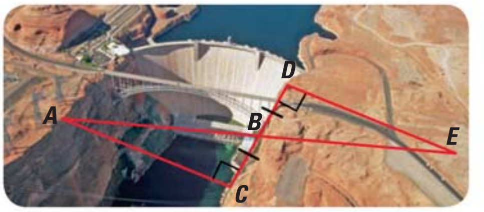
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 2
on p. 257
for Ex. 28

28. CANYON *Explain* how you can find the distance across the canyon.



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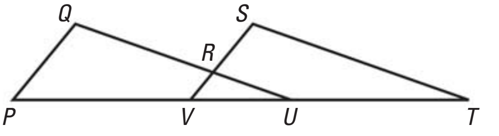
29. PROOF Use the given information and the diagram to write a two-column proof.

GIVEN ▶ $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$

PROVE ▶ $\angle Q \cong \angle S$



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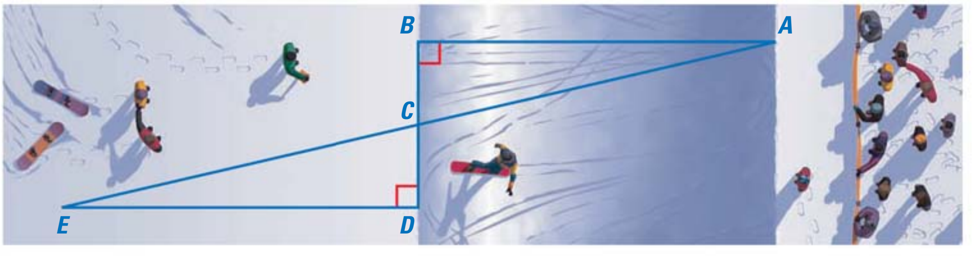
30. SNOWBOARDING In the diagram of the half pipe below, C is the midpoint of $\overline{BD}$. If $EC \approx 11.5$ m, and $CD \approx 2.5$ m, find the approximate distance across the half pipe. *Explain* your reasoning.



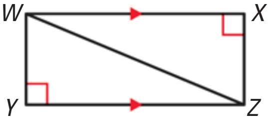
31. ★ **MULTIPLE CHOICE** Using the information in the diagram, you can prove that $\overline{WY} \cong \overline{ZX}$. Which reason would *not* appear in the proof?

Ⓐ SAS Congruence Postulate

Ⓑ AAS Congruence Theorem

Ⓒ Alternate Interior Angles Theorem

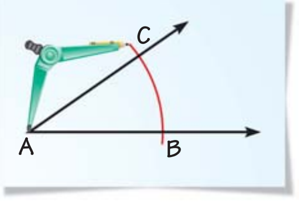
Ⓓ Right Angle Congruence Theorem



EXAMPLE 4
on p. 258
for Ex. 32

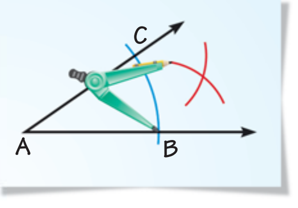
32. PROVING A CONSTRUCTION The diagrams below show the construction on page 34 used to bisect $\angle A$. By construction, you can assume that $\overline{AB} \cong \overline{AC}$ and $\overline{BG} \cong \overline{CG}$. Write a proof to verify that $\overrightarrow{AG}$ bisects $\angle A$.

STEP 1



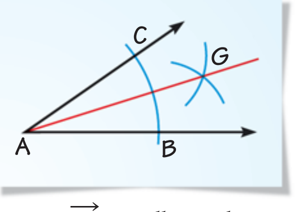
First draw an arc with center A. Label the points where the arc intersects the sides of the angle points B and C.

STEP 2



Draw an arc with center C. Using the same radius, draw an arc with center B. Label the intersection point G.

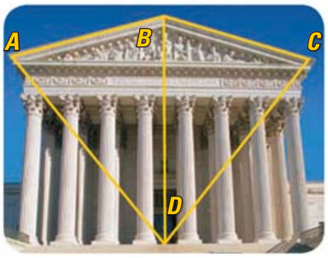
STEP 3



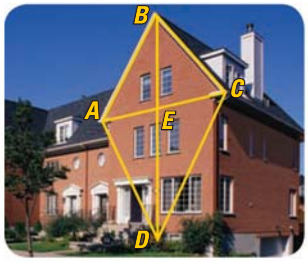
Draw $\overrightarrow{AG}$. It follows that $\angle BAG \cong \angle CAG$.

ARCHITECTURE **Can you use the given information to determine that $\overline{AB} \cong \overline{BC}$? *Justify* your answer.**

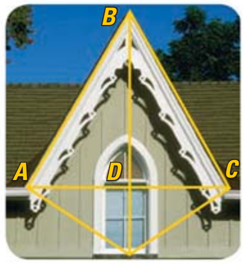
33. $\angle ABD \cong \angle CBD$, $AD = CD$



34. $\overline{AC} \perp \overline{BD}$, $\triangle ADE \cong \triangle CDE$

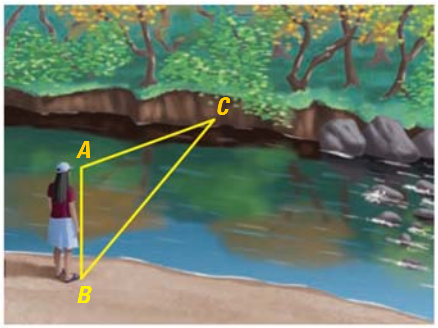


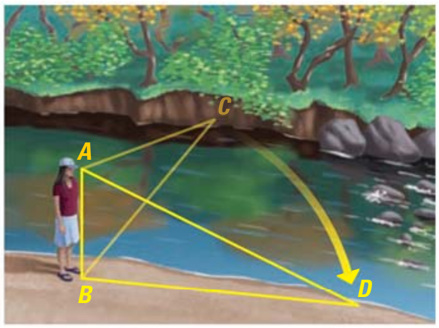
35. $\overline{BD}$ bisects $\overline{AC}$, $\overline{AD} \perp \overline{BD}$



36. ★ **EXTENDED RESPONSE** You can use the method described below to find the distance across a river. You will need a cap with a visor.

- Stand on one side of the river and look straight across to a point on the other side. Align the visor of your cap with that point.
- Without changing the inclination of your neck and head, turn sideways until the visor is in line with a point on your side of the stream.
- Measure the distance *BD* between your feet and that point.



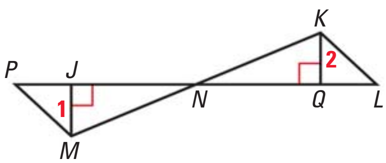


a. What corresponding parts of the two triangles can you assume are congruent? What postulate or theorem can you use to show that the two triangles are congruent?

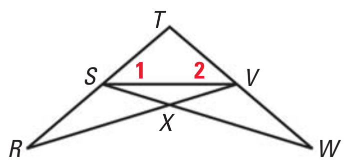
b. *Explain* why *BD* is also the distance across the stream.

PROOF **Use the given information and the diagram to prove that $\angle 1 \cong \angle 2$.**

37. **GIVEN** ▶ $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$



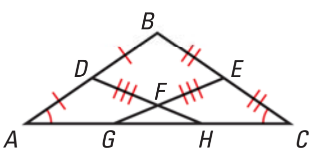
38. **GIVEN** ▶ $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$



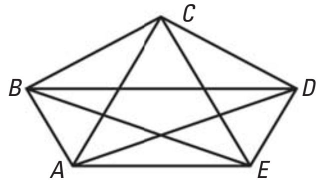
39. **PROOF** Write a proof.

GIVEN ▶ $\overline{BA} \cong \overline{BC}$, *D* and *E* are midpoints, $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$

PROVE ▶ $\overline{FG} \cong \overline{FH}$

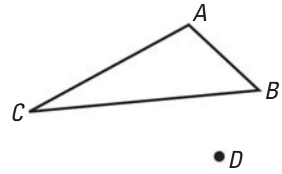


40. CHALLENGE In the diagram of pentagon $ABCDE$, $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, and $\overline{AC} \cong \overline{EC}$. Write a proof that shows $\overline{AD} \cong \overline{EB}$.



MIXED REVIEW

How many lines can be drawn that fit each description? Copy the diagram and sketch all the lines. *(p. 147)*



41. Line(s) through B and parallel to $\overleftrightarrow{AC}$
42. Line(s) through A and perpendicular to $\overleftrightarrow{BC}$
43. Line(s) through D and C

PREVIEW
Prepare for Lesson 4.7 in Exs. 44–46.

The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

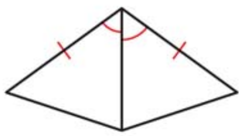
44. $m\angle A = x°$
$m\angle B = (4x)°$
$m\angle C = (5x)°$

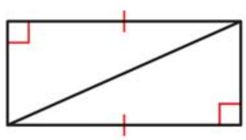
45. $m\angle A = x°$
$m\angle B = (5x)°$
$m\angle C = (x + 19)°$

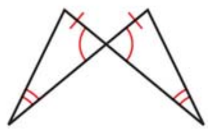
46. $m\angle A = (x - 22)°$
$m\angle B = (x + 16)°$
$m\angle C = (2x - 14)°$

QUIZ *for Lessons 4.4–4.6*

Decide which method, SAS, ASA, AAS, or HL, can be used to prove that the triangles are congruent. *(pp. 240, 249)*

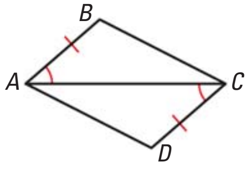
1. 

2. 

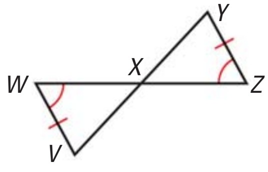
3. 

Use the given information to write a proof.

4. **GIVEN** ▶ $\angle BAC \cong \angle DCA$, $\overline{AB} \cong \overline{CD}$
PROVE ▶ $\triangle ABC \cong \triangle CDA$ *(p. 240)*

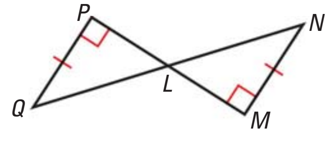


5. **GIVEN** ▶ $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$
PROVE ▶ $\triangle VWX \cong \triangle YZX$ *(p. 249)*



6. Write a plan for a proof. *(p. 256)*

GIVEN ▶ $\overline{PQ} \cong \overline{MN}$, $m\angle P = m\angle M = 90°$
PROVE ▶ $\overline{QL} \cong \overline{NL}$



4.7 Use Isosceles and Equilateral Triangles

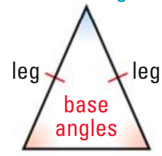
Before You learned about isosceles and equilateral triangles.

Now You will use theorems about isosceles and equilateral triangles.

Why? So you can solve a problem about architecture, as in Ex. 40.

Key Vocabulary
- **legs**
- **vertex angle**
- **base**
- **base angles**

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



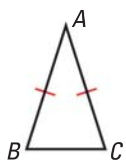
THEOREMS *For Your Notebook*

THEOREM 4.7 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof: p. 265

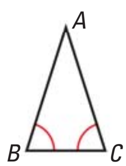


THEOREM 4.8 Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

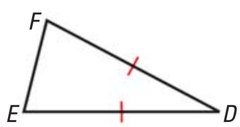
If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof: Ex. 45, p. 269



EXAMPLE 1 Apply the Base Angles Theorem

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.



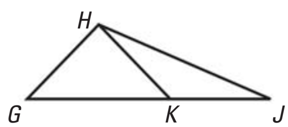
Solution

▶ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

✓ GUIDED PRACTICE for Example 1

Copy and complete the statement.

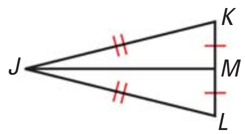
1. If $\overline{HG} \cong \overline{HK}$, then $\angle$ _?_ $\cong \angle$ _?_ .
2. If $\angle KHJ \cong \angle KJH$, then _?_ $\cong$ _?_ .



PROOF Base Angles Theorem

GIVEN ▶ $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\angle K \cong \angle L$



Plan for Proof

a. Draw $\overline{JM}$ so that it bisects $\overline{KL}$.

b. Use SSS to show that $\triangle JMK \cong \triangle JML$.

c. Use properties of congruent triangles to show that $\angle K \cong \angle L$.

Plan in Action

	STATEMENTS	REASONS
	1. M is the midpoint of $\overline{KL}$.	1. Definition of midpoint
a.	2. Draw $\overline{JM}$.	2. Two points determine a line.
	3. $\overline{MK} \cong \overline{ML}$	3. Definition of midpoint
	4. $\overline{JK} \cong \overline{JL}$	4. Given
	5. $\overline{JM} \cong \overline{JM}$	5. Reflexive Property of Congruence
b.	6. $\triangle JMK \cong \triangle JML$	6. SSS Congruence Postulate
c.	7. $\angle K \cong \angle L$	7. Corresp. parts of $\cong$ ⚠ are $\cong$.

Recall that an equilateral triangle has three congruent sides.

WRITE A BICONDITIONAL
The corollaries state that a triangle is *equilateral* if and only if it is *equiangular.*

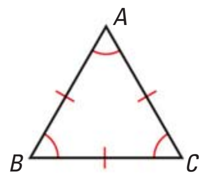
COROLLARIES *For Your Notebook*

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

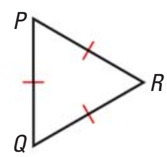
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Find measures in a triangle

Find the measures of $\angle P$, $\angle Q$, and $\angle R$.



The diagram shows that $\triangle PQR$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180°$ **Triangle Sum Theorem**

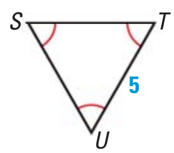
$m\angle P = 60°$ **Divide each side by 3.**

▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

✓ GUIDED PRACTICE for Example 2

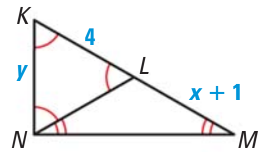
3. Find ST in the triangle at the right.

4. Is it possible for an equilateral triangle to have an angle measure other than 60°? *Explain.*



EXAMPLE 3 Use isosceles and equilateral triangles

xy ALGEBRA Find the values of x and y in the diagram.



Solution

STEP 1 **Find** the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. Therefore, $y = 4$.

AVOID ERRORS
You cannot use $\angle N$ to refer to $\angle LNM$ because three angles have N as their vertex.

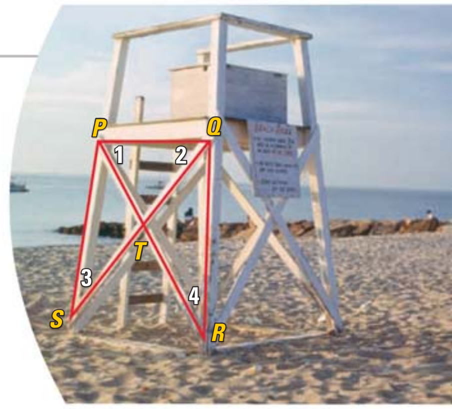
STEP 2 **Find** the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$ and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$ — **Definition of congruent segments**

$4 = x + 1$ — **Substitute 4 for LN and $x + 1$ for LM.**

$3 = x$ — **Subtract 1 from each side.**

EXAMPLE 4 Solve a multi-step problem



LIFEGUARD TOWER In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

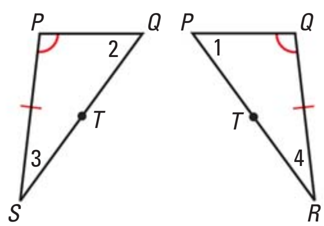
a. What congruence postulate can you use to prove that $\triangle QPS \cong \triangle PQR$?

b. Explain why $\triangle PQT$ is isosceles.

c. Show that $\triangle PTS \cong \triangle QTR$.

Solution

AVOID ERRORS
When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

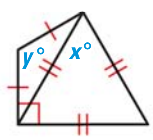


a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Postulate, $\triangle QPS \cong \triangle PQR$.

b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresp. parts of $\cong$ △s are $\cong$. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

c. You know that $\overline{PS} \cong \overline{QR}$, and $\angle 3 \cong \angle 4$ because corresp. parts of $\cong$ △s are $\cong$. Also, $\angle PTS \cong \angle QTR$ by the Vertical Angles Congruence Theorem. So, $\triangle PTS \cong \triangle QTR$ by the AAS Congruence Theorem.

✓ GUIDED PRACTICE for Examples 3 and 4



5. Find the values of x and y in the diagram.

6. **REASONING** Use parts (b) and (c) in Example 4 and the SSS Congruence Postulate to give a different proof that $\triangle QPS \cong \triangle PQR$.

4.7 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 17, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 19, 30, 31, 42, and 46

SKILL PRACTICE

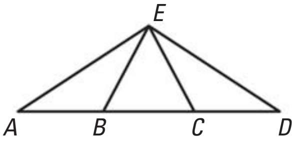
1. **VOCABULARY** Define the *vertex angle* of an isosceles triangle.

2. ★ **WRITING** What is the relationship between the base angles of an isosceles triangle? *Explain.*

EXAMPLE 1 on p. 264 for Exs. 3–6

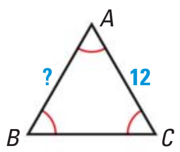
USING DIAGRAMS In Exercises 3–6, use the diagram. Copy and complete the statement. Tell what theorem you used.

3. If $\overline{AE} \cong \overline{DE}$, then $\angle$ _?_ $\cong \angle$ _?_.
4. If $\overline{AB} \cong \overline{EB}$, then $\angle$ _?_ $\cong \angle$ _?_.
5. If $\angle D \cong \angle CED$, then _?_ $\cong$ _?_.
6. If $\angle EBC \cong \angle ECB$, then _?_ $\cong$ _?_.



EXAMPLE 2 on p. 265 for Exs. 7–14

REASONING Find the unknown measure.

7.


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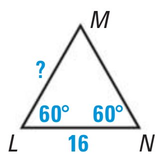

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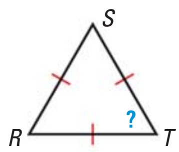
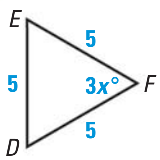


10. **DRAWING DIAGRAMS** A base angle in an isosceles triangle measures 37°. Draw and label the triangle. What is the measure of the vertex angle?

ALGEBRA Find the value of *x*.

11.


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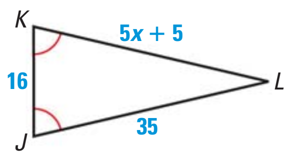

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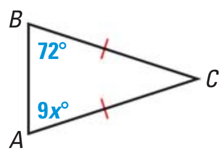
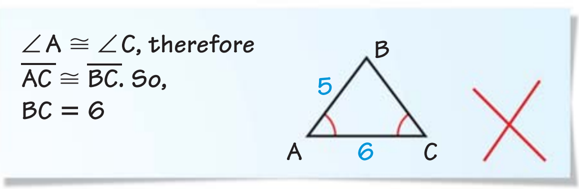
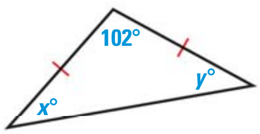


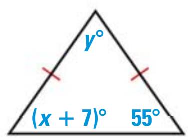
14. **ERROR ANALYSIS** *Describe* and correct the error made in finding *BC* in the diagram shown.

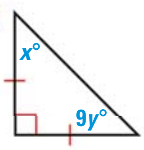


EXAMPLE 3 on p. 266 for Exs. 15–17

ALGEBRA Find the values of *x* and *y*.

15.


16.


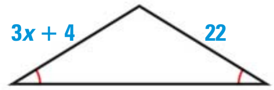
17.


18. ★ **SHORT RESPONSE** Are isosceles triangles always acute triangles? *Explain* your reasoning.

19. ★ **MULTIPLE CHOICE** What is the value of x in the diagram?

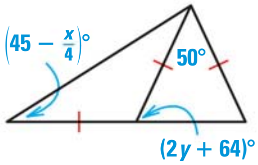
(A) 5 (B) 6

(C) 7 (D) 9

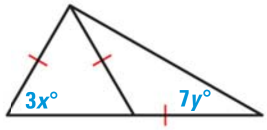


ALGEBRA **Find the values of x and y, if possible. *Explain* your reasoning.**

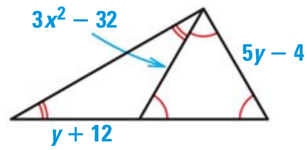
20.



21.

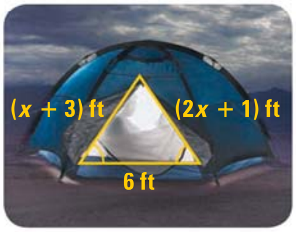


22.

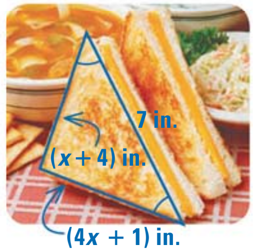


ALGEBRA **Find the perimeter of the triangle.**

23.



24.



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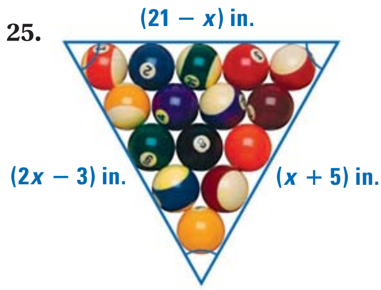


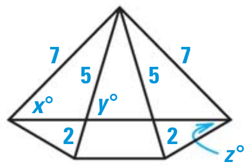
REASONING **In Exercises 26–29, use the diagram. State whether the given values for x, y, and z are possible or not. If not, *explain*.**

26. $x = 90, y = 68, z = 42$

27. $x = 40, y = 72, z = 36$

28. $x = 25, y = 25, z = 15$

29. $x = 42, y = 72, z = 33$

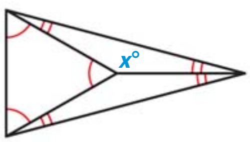


30. ★ **SHORT RESPONSE** In $\triangle DEF$, $m\angle D = (4x + 2)°$, $m\angle E = (6x - 30)°$, and $m\angle F = 3x°$. What type of triangle is $\triangle DEF$? *Explain* your reasoning.

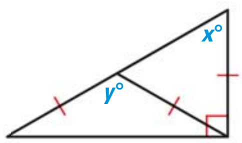
31. ★ **SHORT RESPONSE** In $\triangle ABC$, D is the midpoint of $\overline{AC}$, and $\overline{BD}$ is perpendicular to $\overline{AC}$. *Explain* why $\triangle ABC$ is isosceles.

ALGEBRA **Find the value(s) of the variable(s). *Explain* your reasoning.**

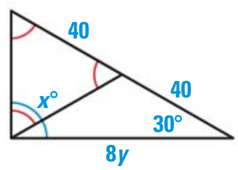
32.



33.



34.



35. **REASONING** The measure of an exterior angle of an isosceles triangle is 130°. What are the possible angle measures of the triangle? *Explain*.

36. **PROOF** Let $\triangle ABC$ be isosceles with vertex angle $\angle A$. Suppose $\angle A$, $\angle B$, and $\angle C$ have integer measures. Prove that $m\angle A$ must be even.

37. **CHALLENGE** The measure of an exterior angle of an isosceles triangle is $x°$. What are the possible angle measures of the triangle in terms of x? *Describe* all the possible values of x.

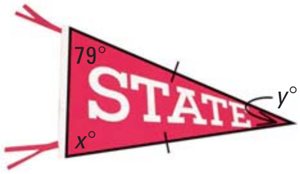
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

38. SPORTS The dimensions of a sports pennant are given in the diagram. Find the values of x and y.

@HomeTutor for problem solving help at classzone.com



39. ADVERTISING A logo in an advertisement is an equilateral triangle with a side length of 5 centimeters. Sketch the logo and give the measure of each side and angle.

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40. ARCHITECTURE The Transamerica Pyramid building shown in the photograph has four faces shaped like isosceles triangles. The measure of a base angle of one of these triangles is about 85°. What is the approximate measure of the vertex angle of the triangle?

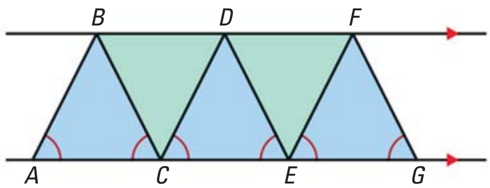
EXAMPLE 4 on p. 266 for Exs. 41–42

41. MULTI-STEP PROBLEM To make a zig-zag pattern, a graphic designer sketches two parallel line segments. Then the designer draws blue and green triangles as shown below.

a. Prove that $\triangle ABC \cong \triangle BCD$.

b. Name all the isosceles triangles in the diagram.

c. Name four angles that are congruent to $\angle ABC$.



42. ★ **VISUAL REASONING** In the pattern below, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle				
Area	1 square unit	?	?	?

a. Reasoning *Explain* how you know that any triangle made out of equilateral triangles will be an equilateral triangle.

b. Area Find the areas of the first four triangles in the pattern.

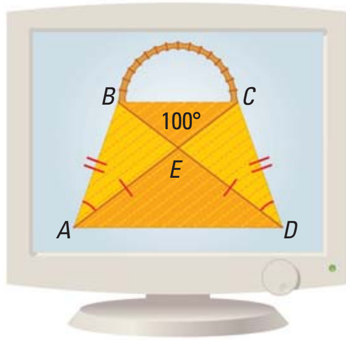
c. Make a Conjecture *Describe* any patterns in the areas. Predict the area of the seventh triangle in the pattern. *Explain* your reasoning.

43. REASONING Let $\triangle PQR$ be an isosceles right triangle with hypotenuse $\overline{QR}$. Find $m\angle P$, $m\angle Q$, and $m\angle R$.

44. REASONING *Explain* how the Corollary to the Base Angles Theorem follows from the Base Angles Theorem.

45. PROVING THEOREM 4.8 Write a proof of the Converse of the Base Angles Theorem.

46. ★ **EXTENDED RESPONSE** Sue is designing fabric purses that she will sell at the school fair. Use the diagram of one of her purses.



a. Prove that $\triangle ABE \cong \triangle DCE$.

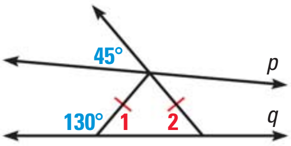
b. Name the isosceles triangles in the purse.

c. Name three angles that are congruent to $\angle EAD$.

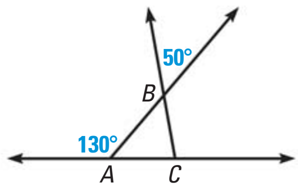
d. **What If?** If the measure of $\angle BEC$ changes, does your answer to part (c) change? *Explain.*

REASONING FROM DIAGRAMS **Use the information in the diagram to answer the question. *Explain* your reasoning.**

47. Is $p \parallel q$?



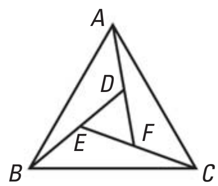
48. Is $\triangle ABC$ isosceles?



49. **PROOF** Write a proof.

GIVEN ▶ $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$.

PROVE ▶ $\triangle DEF$ is equilateral.



50. **COORDINATE GEOMETRY** The coordinates of two vertices of $\triangle TUV$ are $T(0, 4)$ and $U(4, 0)$. *Explain* why the triangle will always be an isosceles triangle if V is any point on the line $y = x$ except $(2, 2)$.

51. **CHALLENGE** The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. *Explain.*

MIXED REVIEW

What quadrant contains the point? *(p. 878)*

52. $(-1, -3)$ **53.** $(-2, 4)$ **54.** $(5, -2)$

Copy and complete the given function table. *(p. 884)*

55.

x	−7	0	5
$y = x - 4$	?	?	?

56.

?	−2	0	1
?	−6	0	3

PREVIEW
Prepare for Lesson 4.8 in Exs. 57–60.

Use the Distance Formula to decide whether $\overline{AB} \cong \overline{AC}$. *(p. 15)*

57. $A(0, 0)$, $B(-5, -6)$, $C(6, 5)$

58. $A(3, -3)$, $B(0, 1)$, $C(-1, 0)$

59. $A(0, 1)$, $B(4, 7)$, $C(-6, 3)$

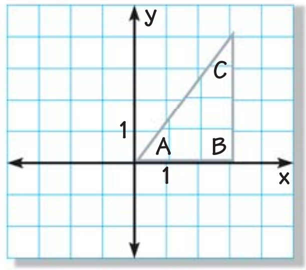
60. $A(-3, 0)$, $B(2, 2)$, $C(2, -2)$

4.8 Investigate Slides and Flips

MATERIALS • graph paper • pencil

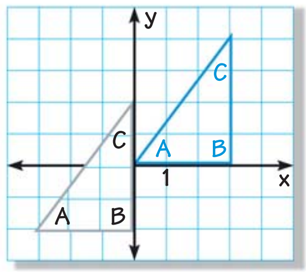
QUESTION **What happens when you slide or flip a triangle?**

EXPLORE 1 Slide a triangle



STEP 1 ***Draw a triangle*** Draw a scalene right triangle with legs of length 3 units and 4 units on a piece of graph paper. Cut out the triangle.

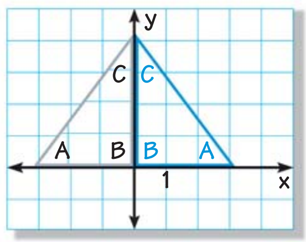
STEP 2 ***Draw coordinate plane*** Draw axes on the graph paper. Place the cut-out triangle so that the coordinates of the vertices are integers. Trace around the triangle and label the vertices.



STEP 3 ***Slide triangle*** Slide the cut-out triangle so it moves left and down. Write a description of the *transformation* and record ordered pairs in a table like the one shown. Repeat this step three times, sliding the triangle left or right *and* up or down to various places in the coordinate plane.

Slide 2 units left and 3 units down.		
Vertex	**Original position**	**New position**
A	(0, 0)	(−3, −2)
B	(3, 0)	(0, −2)
C	(3, 4)	(0, 2)

EXPLORE 2 Flip a triangle



STEP 1 ***Draw a coordinate plane*** Draw and label a second coordinate plane. Place the cut-out triangle so that one vertex is at the origin and one side is along the y-axis, as shown.

STEP 2 ***Flip triangle*** Flip the cut-out triangle over the y-axis. Record a description of the *transformation* and record the ordered pairs in a table. Repeat this step, flipping the triangle over the x-axis.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. How are the coordinates of the original position of the triangle related to the new position in a slide? in a flip?
2. Is the original triangle congruent to the new triangle in a slide? in a flip? *Explain* your reasoning.

4.8 Perform Congruence Transformations

Before	You determined whether two triangles are congruent.
Now	You will create an image congruent to a given triangle.
Why	So you can describe chess moves, as in Ex. 41.

Key Vocabulary
- **transformation**
- **image**
- **translation**
- **reflection**
- **rotation**
- **congruence transformation**

A **transformation** is an operation that moves or changes a geometric figure in some way to produce a new figure. The new figure is called the **image.** A transformation can be shown using an arrow.

The order of the vertices in the transformation statement tells you that P is the image of A, Q is the image of B, and R is the image of C.

$\triangle ABC \rightarrow \triangle PQR$

Original figure   Image

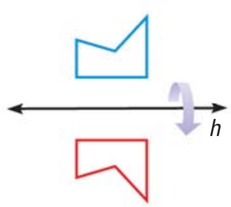
There are three main types of transformations. A **translation** moves every point of a figure the same distance in the same direction. A **reflection** uses a *line of reflection* to create a mirror image of the original figure. A **rotation** turns a figure about a fixed point, called the *center of rotation*.

EXAMPLE 1 Identify transformations

TRANSFORMATIONS You will learn more about transformations in Lesson 6.7 and in Chapter 9.

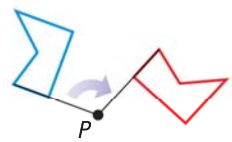
Name the type of transformation demonstrated in each picture.

a.



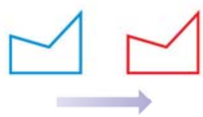
Reflection in a horizontal line

b.



Rotation about a point

c.



Translation in a straight path

✓ GUIDED PRACTICE for Example 1

1. Name the type of transformation shown.

CONGRUENCE Translations, reflections, and rotations are three types of *congruence transformations*. A **congruence transformation** changes the position of the figure without changing its size or shape.

TRANSLATIONS In a coordinate plane, a translation moves an object a given distance right or left and up or down. You can use coordinate notation to describe a translation.

READ DIAGRAMS
In this book, the original figure is blue and the transformation of the figure is red, unless otherwise stated.

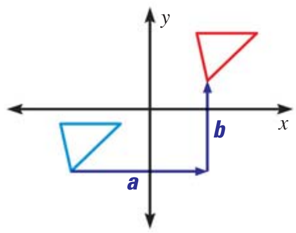
KEY CONCEPT — *For Your Notebook*

Coordinate Notation for a Translation

You can describe a translation by the notation

$$(x, y) \rightarrow (x + a, y + b)$$

which shows that each point (x, y) of the blue figure is translated horizontally a units and vertically b units.



EXAMPLE 2 Translate a figure in the coordinate plane

Figure $ABCD$ has the vertices $A(-4, 3)$, $B(-2, 4)$, $C(-1, 1)$, and $D(-3, 1)$. Sketch $ABCD$ and its image after the translation $(x, y) \rightarrow (x + 5, y - 2)$.

Solution

First draw $ABCD$. Find the translation of each vertex by adding 5 to its x-coordinate and subtracting 2 from its y-coordinate. Then draw $ABCD$ and its image.

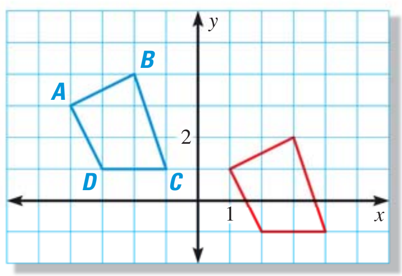
$(x, y) \rightarrow (x + 5, y - 2)$

$A(-4, 3) \rightarrow (1, 1)$

$B(-2, 4) \rightarrow (3, 2)$

$C(-1, 1) \rightarrow (4, -1)$

$D(-3, 1) \rightarrow (2, -1)$

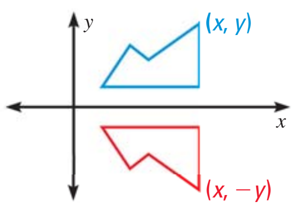


REFLECTIONS In this lesson, when a reflection is shown in a coordinate plane, the line of reflection is always the x-axis or the y-axis.

KEY CONCEPT — *For Your Notebook*

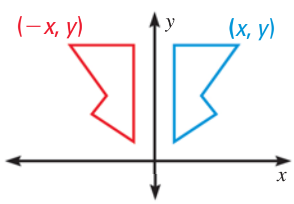
Coordinate Notation for a Reflection

Reflection in the x-axis



Multiply the y-coordinate by −1.
$(x, y) \rightarrow (x, -y)$

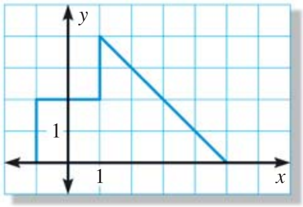
Reflection in the y-axis



Multiply the x-coordinate by −1.
$(x, y) \rightarrow (-x, y)$

EXAMPLE 3 Reflect a figure in the *y*-axis

WOODWORK You are drawing a pattern for a wooden sign. Use a reflection in the *x*-axis to draw the other half of the pattern.



Solution

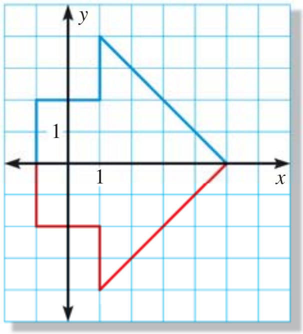
Multiply the *y*-coordinate of each vertex by −1 to find the corresponding vertex in the image.

$(x, y) \rightarrow (x, -y)$

$(-1, 0) \rightarrow (-1, 0)$ $(-1, 2) \rightarrow (-1, -2)$

$(1, 2) \rightarrow (1, -2)$ $(1, 4) \rightarrow (1, -4)$

$(5, 0) \rightarrow (5, 0)$



Use the vertices to draw the image. You can check your results by looking to see if each original point and its image are the same distance from the *x*-axis.

Animated Geometry at classzone.com

GUIDED PRACTICE for Examples 2 and 3

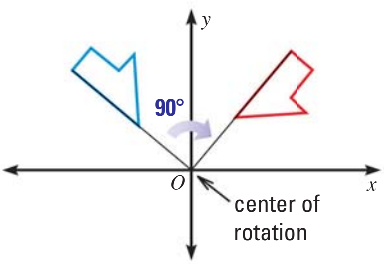
2. The vertices of $\triangle ABC$ are $A(1, 2)$, $B(0, 0)$, and $C(4, 0)$. A translation of $\triangle ABC$ results in the image $\triangle DEF$ with vertices $D(2, 1)$, $E(1, -1)$, and $F(5, -1)$. *Describe* the translation in words and in coordinate notation.

3. The endpoints of $\overline{RS}$ are $R(4, 5)$ and $S(1, -3)$. A reflection of $\overline{RS}$ results in the image $\overline{TU}$, with coordinates $T(4, -5)$ and $U(1, 3)$. Tell which axis $\overline{RS}$ was reflected in and write the coordinate rule for the reflection.

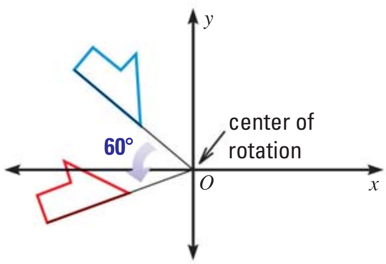
ROTATIONS In this lesson, if a rotation is shown in a coordinate plane, the center of rotation is the origin.

The direction of rotation can be either *clockwise* or *counterclockwise*. The *angle of rotation* is formed by rays drawn from the center of rotation through corresponding points on the original figure and its image.

90° clockwise rotation



60° counterclockwise rotation



Notice that rotations preserve distances from the center of rotation. So, segments drawn from the center of rotation to corresponding points on the figures are congruent.

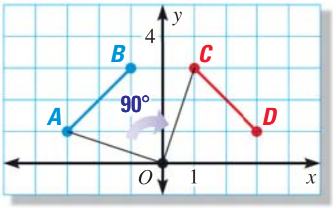
EXAMPLE 4 Identify a rotation

Graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.

a. $A(-3, 1), B(-1, 3), C(1, 3), D(3, 1)$ **b.** $A(0, 1), B(1, 3), C(-1, 1), D(-3, 2)$

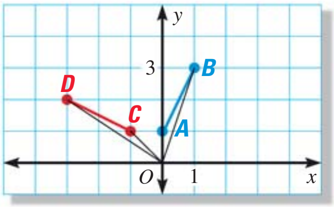
Solution

a.



$m\angle AOC = m\angle BOD = 90°$
This is a 90° clockwise rotation.

b.



$m\angle AOC < m\angle BOD$
This is not a rotation.

EXAMPLE 5 Verify congruence

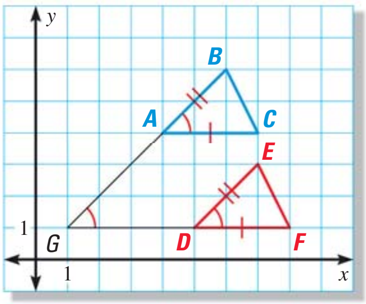
The vertices of $\triangle ABC$ are $A(4, 4)$, $B(6, 6)$, and $C(7, 4)$. The notation $(x, y) \to (x + 1, y - 3)$ describes the translation of $\triangle ABC$ to $\triangle DEF$. Show that $\triangle ABC \cong \triangle DEF$ to verify that the translation is a congruence transformation.

Solution

S You can see that $AC = DF = 3$, so $\overline{AC} \cong \overline{DF}$.

A Using the slopes, $\overline{AB} \parallel \overline{DE}$ and $\overline{AC} \parallel \overline{DF}$. If you extend $\overline{AB}$ and $\overline{DF}$ to form $\angle G$, the Corresponding Angles Postulate gives you $\angle BAC \cong \angle G$ and $\angle G \cong \angle EDF$. Then, $\angle BAC \cong \angle EDF$ by the Transitive Property of Congruence.

S Using the Distance Formula, $AB = DE = 2\sqrt{2}$ so $\overline{AB} \cong \overline{DE}$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate.

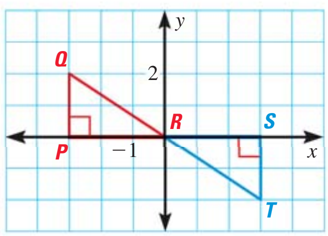


▶ Because $\triangle ABC \cong \triangle DEF$, the translation is a congruence transformation.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Tell whether $\triangle PQR$ is a rotation of $\triangle STR$. If so, give the angle and direction of rotation.

5. Show that $\triangle PQR \cong \triangle STR$ to verify that the transformation is a congruence transformation.



4.8 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 23, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 40, 41, and 43

SKILL PRACTICE

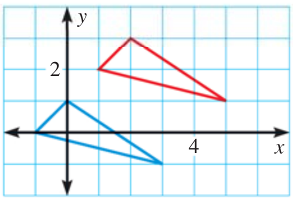
1. VOCABULARY *Describe* the translation $(x, y) \to (x - 1, y + 4)$ in words.

2. ★ WRITING *Explain* why the term *congruence transformation* is used in describing translations, reflections, and rotations.

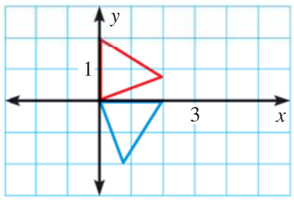
EXAMPLE 1 on p. 272 for Exs. 3–8

IDENTIFYING TRANSFORMATIONS **Name the type of transformation shown.**

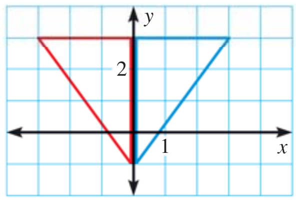
3.



4.



5.



WINDOWS **Decide whether the moving part of the window is a translation.**

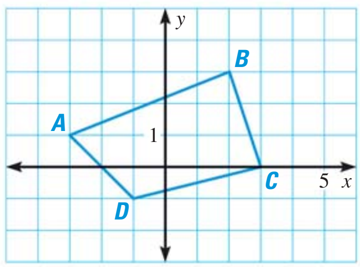
6. Double hung

7. Casement

8. Sliding

EXAMPLE 2 on p. 273 for Exs. 9–16

DRAWING A TRANSLATION **Copy figure *ABCD* and draw its image after the translation.**



9. $(x, y) \to (x + 2, y - 3)$

10. $(x, y) \to (x - 1, y - 5)$

11. $(x, y) \to (x + 4, y + 1)$

12. $(x, y) \to (x - 2, y + 3)$

COORDINATE NOTATION **Use coordinate notation to *describe* the translation.**

13. 4 units to the left, 2 units down

14. 6 units to the right, 3 units up

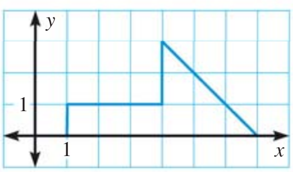
15. 2 units to the right, 1 unit down

16. 7 units to the left, 9 units up

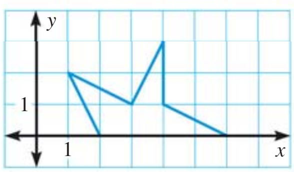
EXAMPLE 3 on p. 274 for Exs. 17–19

DRAWING **Use a reflection in the x-axis to draw the other half of the figure.**

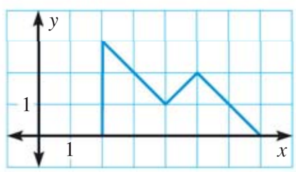
17.



18.



19.



EXAMPLE 4
on p. 275
for Exs. 20–23

ROTATIONS **Use the coordinates to graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.**

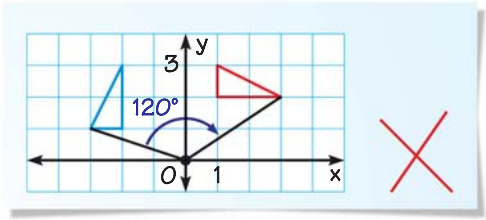
20. $A(1, 2)$, $B(3, 4)$, $C(2, -1)$, $D(4, -3)$

21. $A(-2, -4)$, $B(-1, -2)$, $C(4, 3)$, $D(2, 1)$

22. $A(-4, 0)$, $B(4, -4)$, $C(4, 4)$, $D(0, 4)$

23. $A(1, 2)$, $B(3, 0)$, $C(2, -1)$, $D(2, -3)$

24. **ERROR ANALYSIS** A student says that the red triangle is a 120° clockwise rotation of the blue triangle about the origin. *Describe* and correct the error.



25. ★ **WRITING** Can a point or a line segment be its own image under a transformation? *Explain* and illustrate your answer.

APPLYING TRANSLATIONS **Complete the statement using the description of the translation. In the description, points (0, 3) and (2, 5) are two vertices of a hexagon.**

26. If (0, 3) translates to (0, 0), then (2, 5) translates to __?__.

27. If (0, 3) translates to (1, 2), then (2, 5) translates to __?__.

28. If (0, 3) translates to (−3, −2), then (2, 5) translates to __?__.

xy **ALGEBRA** **A point on an image and the translation are given. Find the corresponding point on the original figure.**

29. Point on image: (4, 0); translation: $(x, y) \rightarrow (x + 2, y - 3)$

30. Point on image: (−3, 5); translation: $(x, y) \rightarrow (-x, y)$

31. Point on image: (6, −9); translation: $(x, y) \rightarrow (x - 7, y - 4)$

32. **CONGRUENCE** Show that the transformation in Exercise 3 is a congruence transformation.

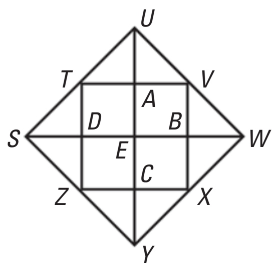
DESCRIBING AN IMAGE **State the segment or triangle that represents the image. You can use tracing paper to help you see the rotation.**

33. 90° clockwise rotation of $\overline{ST}$ about E

34. 90° counterclockwise rotation of $\overline{BX}$ about E

35. 180° rotation of $\triangle BWX$ about E

36. 180° rotation of $\triangle TUA$ about E



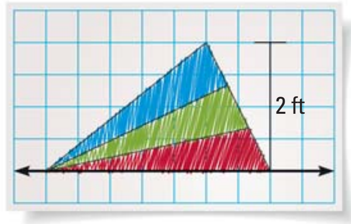
37. **CHALLENGE** Solve for the variables in the transformation of $\overline{AB}$ to $\overline{CD}$ and then to $\overline{EF}$.

$A(2, 3)$, $B(4, 2a)$ → Translation: $(x, y) \rightarrow (x - 2, y + 1)$ → $C(m - 3, 4)$, $D(n - 9, 5)$ → Reflection: in x-axis → $E(0, g - 6)$, $F(8h, -5)$

PROBLEM SOLVING

EXAMPLE 3 on p. 274 for Ex. 38

38. KITES The design for a kite shows the layout and dimensions for only half of the kite.



a. What type of transformation can a designer use to create plans for the entire kite?

b. What is the maximum width of the entire kite?

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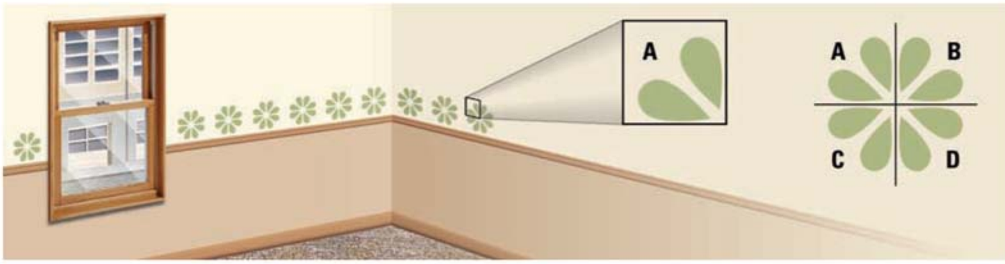
39. STENCILING You are stenciling a room in your home. You want to use the stencil pattern below on the left to create the design shown. Give the angles and directions of rotation you will use to move the stencil from *A* to *B* and from *A* to *C*.



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40. ★ OPEN-ENDED MATH Some words reflect onto themselves through a vertical line of reflection. An example is shown.

a. Find two other words with vertical lines of reflection. Draw the line of reflection for each word.

b. Find two words with horizontal lines of reflection. Draw the line of reflection for each word.

41. ★ SHORT RESPONSE In chess, six different kinds of pieces are moved according to individual rules. The Knight (shaped like a horse) moves in an "L" shape. It moves two squares horizontally or vertically and then one additional square perpendicular to its original direction. When a knight lands on a square with another piece, it *captures* that piece.

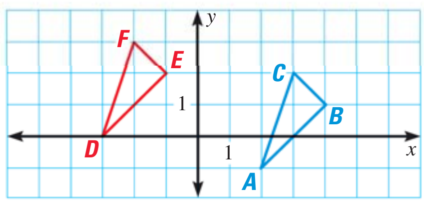
a. *Describe* the translation used by the Black Knight to capture the White Pawn.

b. *Describe* the translation used by the White Knight to capture the Black Pawn.

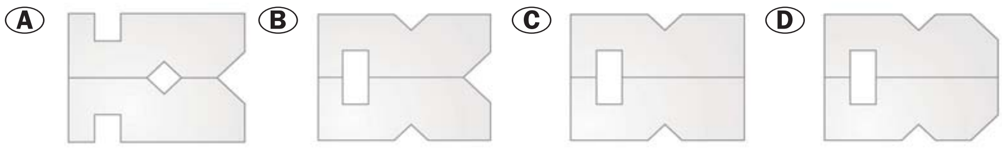
c. After both pawns are captured, can the Black Knight capture the White Knight? *Explain*.

EXAMPLE 5 on p. 275 for Ex. 42

42. VERIFYING CONGRUENCE Show that $\triangle ABC$ and $\triangle DEF$ are right triangles and use the HL Congruence Theorem to verify that $\triangle DEF$ is a congruence transformation of $\triangle ABC$.



43. ★ **MULTIPLE CHOICE** A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the unfolded piece of paper?



44. **CHALLENGE** A triangle is rotated 90° counterclockwise and then translated three units up. The vertices of the final image are $A(-4, 4)$, $B(-1, 6)$, and $C(-1, 4)$. Find the vertices of the original triangle. Would the final image be the same if the original triangle was translated 3 units up and then rotated 90° counterclockwise? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 5.1 in Exs. 45–50.

Simplify the expression. Variables *a* and *b* are positive.

45. $\frac{-a-0}{0-(-b)}$ *(p. 870)* 46. $|(a+b)-a|$ *(p. 870)* 47. $\frac{2a+2b}{2}$ *(p. 139)*

Simplify the expression. Variables *a* and *b* are positive. *(p. 139)*

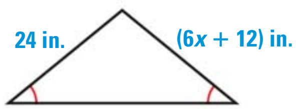
48. $\sqrt{(-b)^2}$ 49. $\sqrt{(2a)^2}$ 50. $\sqrt{(2a-a)^2+(0-b)^2}$

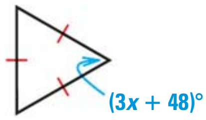
51. Use the SSS Congruence Postulate to show $\triangle RST \cong \triangle UVW$. *(p. 234)*

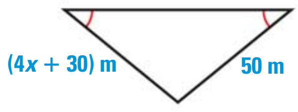
$R(1, -4), S(1, -1), T(6, -1)$ $U(1, 4), V(1, 1), W(6, 1)$

QUIZ for Lessons 4.7–4.8

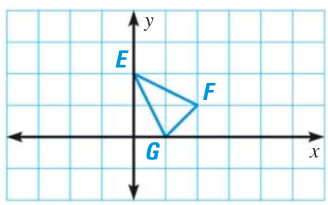
Find the value of *x*. *(p. 264)*

1. 

2. 

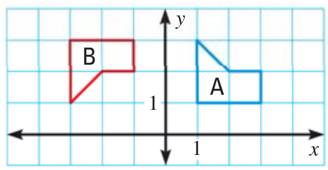
3. 

Copy $\triangle EFG$ and draw its image after the transformation. Identify the type of transformation. *(p. 272)*



4. $(x, y) \to (x+4, y-1)$
5. $(x, y) \to (-x, y)$
6. $(x, y) \to (x, -y)$
7. $(x, y) \to (x-3, y+2)$

8. Is Figure B a rotation of Figure A about the origin? If so, give the angle and direction of rotation. *(p. 272)*



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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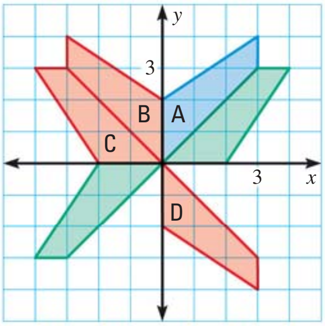
Lessons 4.5–4.8

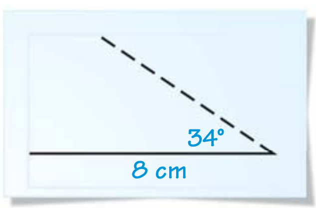
1. **MULTI-STEP PROBLEM** Use the quilt pattern shown below.



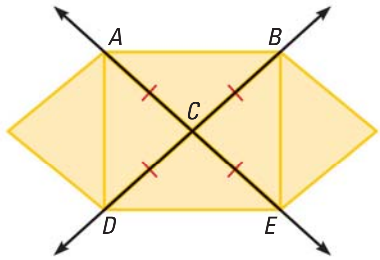
   a. Figure B is the image of Figure A. Name and *describe* the transformation.
   b. Figure C is the image of Figure A. Name and *describe* the transformation.
   c. Figure D is the image of Figure A. Name and *describe* the transformation.
   d. *Explain* how you could complete the quilt pattern using transformations of Figure A.

2. **SHORT RESPONSE** You are told that a triangle has sides that are 5 centimeters and 3 centimeters long. You are also told that the side that is 5 centimeters long forms an angle with the third side that measures 28°. Is there only one triangle that has these given dimensions? *Explain* why or why not.

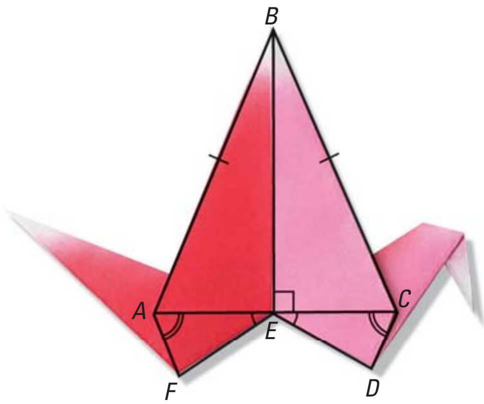
3. **OPEN-ENDED** A friend has drawn a triangle on a piece of paper and she is describing the triangle so that you can draw one that is congruent to hers. So far, she has told you that the length of one side is 8 centimeters and one of the angles formed with this side is 34°. *Describe* three pieces of additional information you could use to construct the triangle.



4. **SHORT RESPONSE** Can the triangles ACD and BCE be proven congruent using the information given in the diagram? Can you show that $\overline{AD} \cong \overline{BE}$? *Explain*.



5. **EXTENDED RESPONSE** Use the information given in the diagram to prove the statements below.



   a. Prove that $\angle BCE \cong \angle BAE$.
   b. Prove that $\overline{AF} \cong \overline{CD}$.

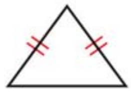
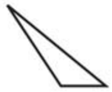
6. **GRIDDED ANSWER** Find the value of x in the diagram.

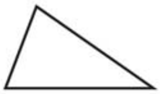
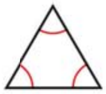
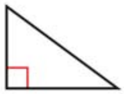
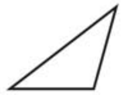
4 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

Big Idea 1

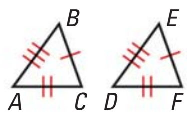
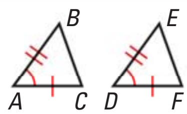
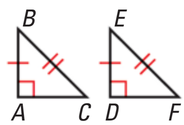
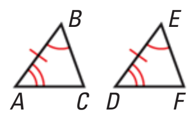
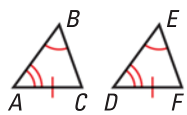
Classifying Triangles by Sides and Angles

	Equilateral	Isosceles	Scalene
Sides	 3 congruent sides	 2 or 3 congruent sides	 No congruent sides

	Acute	Equiangular	Right	Obtuse
Angles	 3 angles $< 90°$	 3 angles $= 60°$	 1 angle $= 90°$	 1 angle $> 90°$

Big Idea 2

Proving That Triangles Are Congruent

SSS	All three sides are congruent.	$\triangle ABC \cong \triangle DEF$	
SAS	Two sides and the included angle are congruent.	$\triangle ABC \cong \triangle DEF$	
HL	The hypotenuse and one of the legs are congruent. (Right triangles only)	$\triangle ABC \cong \triangle DEF$	
ASA	Two angles and the included side are congruent.	$\triangle ABC \cong \triangle DEF$	
AAS	Two angles and a (non-included) side are congruent.	$\triangle ABC \cong \triangle DEF$	

Big Idea 3

Using Coordinate Geometry to Investigate Triangle Relationships

You can use the Distance and Midpoint Formulas to apply postulates and theorems to triangles in the coordinate plane.

4 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- triangle, *p. 217*
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles, *p. 218*
- exterior angles, *p. 218*
- corollary to a theorem, *p. 220*
- congruent figures, *p. 225*
- corresponding parts, *p. 225*
- right triangle, *p. 241*
  legs, hypotenuse
- flow proof, *p. 250*
- isosceles triangle, *p. 264*
  legs, vertex angle, base, base angles
- transformation, *p. 272*
- image, *p. 272*
- congruence transformation, *p. 272*
  translation, reflection, rotation

VOCABULARY EXERCISES

1. Copy and complete: A triangle with three congruent angles is called __?__.
2. **WRITING** *Compare* vertex angles and base angles.
3. **WRITING** *Describe* the difference between isosceles and scalene triangles.
4. Sketch an acute scalene triangle. Label its interior angles 1, 2, and 3. Then draw and shade its exterior angles.
5. If $\triangle PQR \cong \triangle LMN$, which angles are corresponding angles? Which sides are corresponding sides?

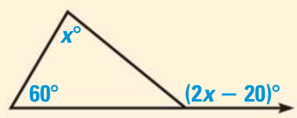
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 4.

4.1 Apply Triangle Sum Properties *pp. 217–224*

EXAMPLE

Find the measure of the exterior angle shown.



Use the Exterior Angle Theorem to write and solve an equation to find the value of x.

$(2x - 20)^\circ = 60^\circ + x^\circ$ **Apply the Exterior Angle Theorem.**

$x = 80$ **Solve for x.**

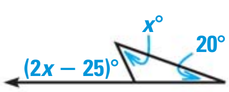
The measure of the exterior angle is $(2 \cdot 80 - 20)^\circ$, or 140°.

EXERCISES

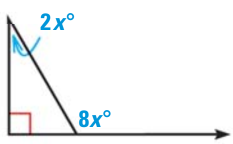
EXAMPLE 3 on p. 219 for Exs. 6–8

Find the measure of the exterior angle shown.

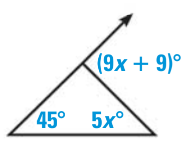
6.



7.



8.



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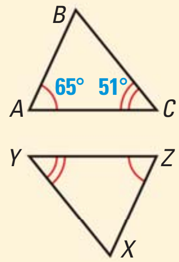
Chapter Review Practice

4.2 Apply Congruence and Triangles

pp. 225–231

EXAMPLE

Use the Third Angles Theorem to find $m\angle X$.



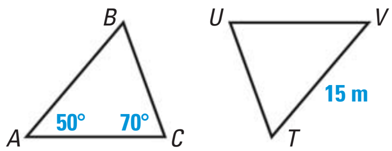
In the diagram, $\angle A \cong \angle Z$ and $\angle C \cong \angle Y$. By the Third Angles Theorem, $\angle B \cong \angle X$. Then by the Triangle Sum Theorem, $m\angle B = 180° - 65° - 51° = 64°$.

So, $m\angle X = m\angle B = 64°$ by the definition of congruent angles.

EXERCISES

EXAMPLES 2 and 4
on pp. 226–227
for Exs. 9–14

In the diagram, $\triangle ABC \cong \triangle VTU$. Find the indicated measure.



9. $m\angle B$

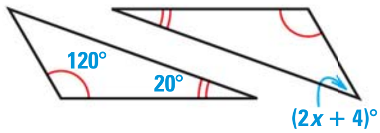
10. AB

11. $m\angle T$

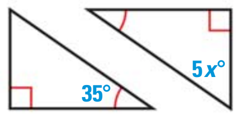
12. $m\angle V$

Find the value of *x*.

13.



14.

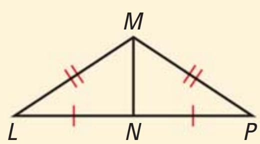


4.3 Prove Triangles Congruent by SSS

pp. 234–239

EXAMPLE

Prove that $\triangle LMN \cong \triangle PMN$.



The marks on the diagram show that $\overline{LM} \cong \overline{PM}$ and $\overline{LN} \cong \overline{PN}$. By the Reflexive Property, $\overline{MN} \cong \overline{MN}$.

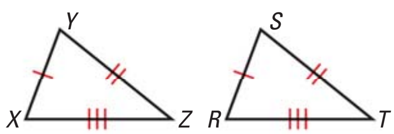
So, by the SSS Congruence Postulate, $\triangle LMN \cong \triangle PMN$.

EXERCISES

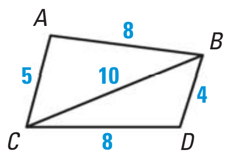
EXAMPLE 1
on p. 234
for Exs. 15–16

Decide whether the congruence statement is true. *Explain* your reasoning.

15. $\triangle XYZ \cong \triangle RST$



16. $\triangle ABC \cong \triangle DCB$

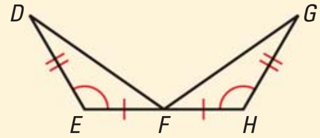


4 CHAPTER REVIEW

4.4 Prove Triangles Congruent by SAS and HL *pp. 240–246*

EXAMPLE

Prove that $\triangle DEF \cong \triangle GHF$.



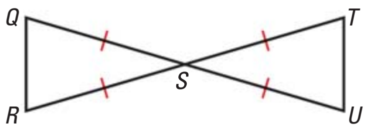
From the diagram, $\overline{DE} \cong \overline{GH}$, $\angle E \cong \angle H$, and $\overline{EF} \cong \overline{HF}$. By the SAS Congruence Postulate, $\triangle DEF \cong \triangle GHF$.

EXERCISES

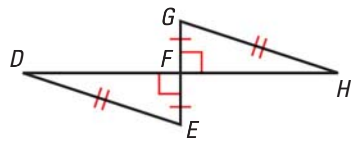
EXAMPLES 1 and 3 on pp. 240, 242 for Exs. 17–18

Decide whether the congruence statement is true. *Explain* your reasoning.

17. $\triangle QRS \cong \triangle TUS$



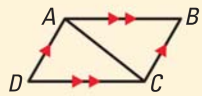
18. $\triangle DEF \cong \triangle GHF$



4.5 Prove Triangles Congruent by ASA and AAS *pp. 249–255*

EXAMPLE

Prove that $\triangle DAC \cong \triangle BCA$.



By the Reflexive Property, $\overline{AC} \cong \overline{AC}$. Because $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, $\angle DAC \cong \angle BCA$ and $\angle DCA \cong \angle BAC$ by the Alternate Interior Angles Theorem. So, by the ASA Congruence Postulate, $\triangle ADC \cong \triangle ABC$.

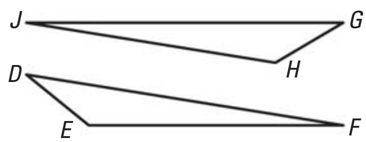
EXERCISES

EXAMPLES 1 and 2 on p. 250 for Exs. 19–20

State the third congruence that is needed to prove that $\triangle DEF \cong \triangle GHJ$ using the given postulate or theorem.

19. **GIVEN** ▶ $\overline{DE} \cong \overline{GH}$, $\angle D \cong \angle G$, __?__ $\cong$ __?__
Use the AAS Congruence Theorem.

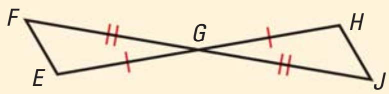
20. **GIVEN** ▶ $\overline{DF} \cong \overline{GJ}$, $\angle F \cong \angle J$, __?__ $\cong$ __?__
Use the ASA Congruence Postulate.



4.6 Use Congruent Triangles *pp. 256–263*

EXAMPLE

GIVEN ▶ $\overline{FG} \cong \overline{JG}$, $\overline{EG} \cong \overline{HG}$
PROVE ▶ $\overline{EF} \cong \overline{HJ}$



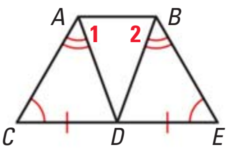
You are given that $\overline{FG} \cong \overline{JG}$ and $\overline{EG} \cong \overline{HG}$. By the Vertical Angles Theorem, $\angle FGE \cong \angle JGH$. So, $\triangle FGE \cong \triangle JGH$ by the SAS Congruence Postulate. Corres. parts of $\cong$ △s are $\cong$, so $\overline{EF} \cong \overline{HJ}$.

EXERCISES

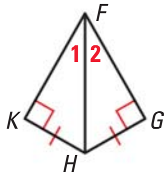
EXAMPLE 3
on p. 257
for Exs. 21–23

Write a plan for proving that $\angle 1 \cong \angle 2$.

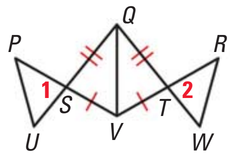
21.



22.



23.

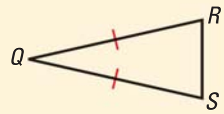


4.7 Use Isosceles and Equilateral Triangles

pp. 264–270

EXAMPLE

$\triangle QRS$ is isosceles. Name two congruent angles.



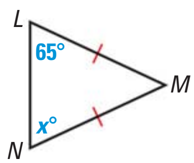
$\overline{QR} \cong \overline{QS}$, so by the Base Angles Theorem, $\angle R \cong \angle S$.

EXERCISES

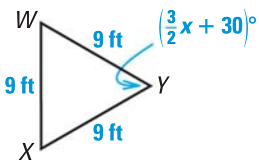
EXAMPLE 3
on p. 266
for Exs. 24–26

Find the value of *x*.

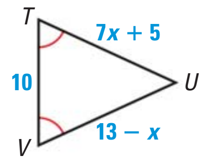
24.



25.



26.



4.8 Perform Congruence Transformations

pp. 272–279

EXAMPLE

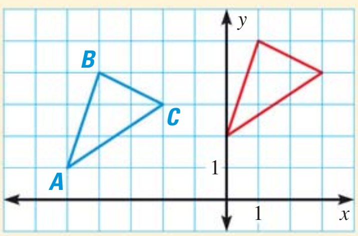
Triangle *ABC* has vertices $A(-5, 1)$, $B(-4, 4)$, and $C(-2, 3)$. Sketch $\triangle ABC$ and its image after the translation $(x, y) \rightarrow (x + 5, y + 1)$.

$(x, y) \rightarrow (x + 5, y + 1)$

$A(-5, 1) \rightarrow (0, 2)$

$B(-4, 4) \rightarrow (1, 5)$

$C(-2, 3) \rightarrow (3, 4)$



EXERCISES

EXAMPLES 2 and 3
on pp. 273–274
for Exs. 27–29

Triangle *QRS* has vertices $Q(2, -1)$, $R(5, -2)$, and $S(2, -3)$. Sketch $\triangle QRS$ and its image after the transformation.

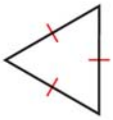
27. $(x, y) \rightarrow (x - 1, y + 5)$

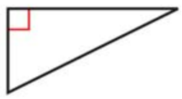
28. $(x, y) \rightarrow (x, -y)$

29. $(x, y) \rightarrow (-x, -y)$

4 CHAPTER TEST

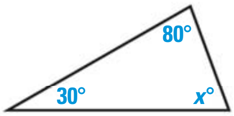
Classify the triangle by its sides and by its angles.

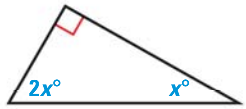
1. 

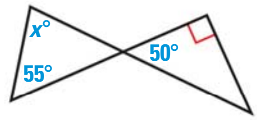
2. 

3. 

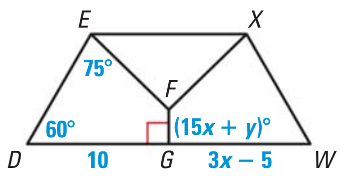
In Exercises 4–6, find the value of *x*.

4. 

5. 

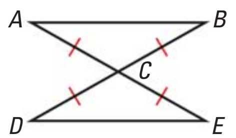
6. 

7. In the diagram, $DEFG \cong WXFG$. Find the values of x and y.

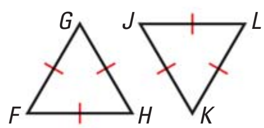


In Exercises 8–10, decide whether the triangles can be proven congruent by the given postulate.

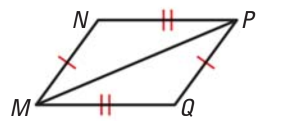
8. $\triangle ABC \cong \triangle EDC$ by SAS



9. $\triangle FGH \cong \triangle JKL$ by ASA



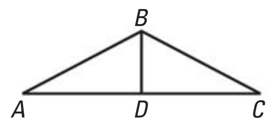
10. $\triangle MNP \cong \triangle PQM$ by SSS



11. Write a proof.

GIVEN ▶ $\triangle ABC$ is isosceles, $\overline{BD}$ bisects $\angle B$.

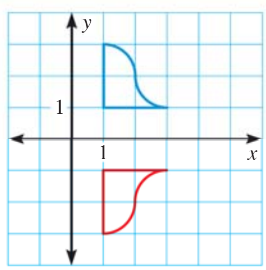
PROVE ▶ $\triangle ABD \cong \triangle CBD$

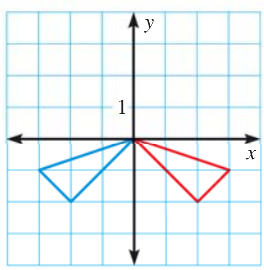


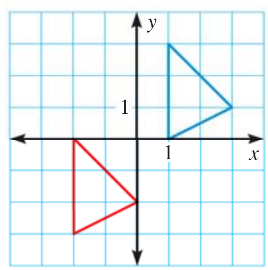
12. What is the third congruence needed to prove that $\triangle PQR \cong \triangle STU$ using the indicated theorem?

a. HL **b.** AAS

Decide whether the transformation is a *translation, reflection*, or *rotation*.

13. 

14. 

15. 

4 ALGEBRA REVIEW

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SOLVE INEQUALITIES AND ABSOLUTE VALUE EQUATIONS

EXAMPLE 1 *Solve inequalities*

Solve $-3x + 7 \le 28$. Then graph the solution.

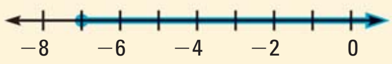
When you multiply or divide each side of an inequality by a *negative* number, you must reverse the inequality symbol to obtain an equivalent inequality.

$-3x + 7 \le 28$ **Write original inequality.**

$-3x \le 21$ **Subtract 7 from both sides.**

$x \ge -7$ **Divide each side by -3. Reverse the inequality symbol.**

▶ The solutions are all real numbers greater than or equal to -7. The graph is shown at the right.



EXAMPLE 2 *Solve absolute value equations*

Solve $|2x + 1| = 5$.

The expression inside the absolute value bars can represent 5 or -5.

STEP 1 Assume $2x + 1$ represents 5.

$2x + 1 = 5$

$2x = 4$

$x = 2$

STEP 2 Assume $2x + 1$ represents -5.

$2x + 1 = -5$

$2x = -6$

$x = -3$

▶ The solutions are 2 and -3.

EXERCISES

EXAMPLE 1 for Exs. 1–12

Solve the inequality. Then graph the solution.

1. $x - 6 > -4$

2. $7 - c \le -1$

3. $-54 \ge 6x$

4. $\frac{5}{2}t + 8 \le 33$

5. $3(y + 2) < 3$

6. $\frac{1}{4}z < 2$

7. $5k + 1 \ge -11$

8. $13.6 > -0.8 - 7.2r$

9. $6x + 7 < 2x - 3$

10. $-v + 12 \le 9 - 2v$

11. $4(n + 5) \ge 5 - n$

12. $5y + 3 \ge 2(y - 9)$

EXAMPLE 2 for Exs. 13–27

Solve the equation.

13. $|x - 5| = 3$

14. $|x + 6| = 2$

15. $|4 - x| = 4$

16. $|2 - x| = 0.5$

17. $|3x - 1| = 8$

18. $|4x + 5| = 7$

19. $|x - 1.3| = 2.1$

20. $|3x - 15| = 0$

21. $|6x - 2| = 4$

22. $|8x + 1| = 17$

23. $|9 - 2x| = 19$

24. $|0.5x - 4| = 2$

25. $|5x - 2| = 8$

26. $|7x + 4| = 11$

27. $|3x - 11| = 4$